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The Dating of Ptolemy's *Almagest* Based on the Coverings of the Stars and on Lunar Eclipses

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Abstract. This paper is a natural extension and continuation of the authors' studies of the astronomical dating problem of Ptolemy's famous *Almagest*. In previous papers, the authors suggested and developed a new geometrical-statistical method for dating ancient star catalogues. This method was then applied to Ptolemy's *Almagest*. The results obtained do not confirm the traditional dating of the *Almagest* (2nd century AD or 2nd century BC) but shift it to the epoch AD 600–1300. In this paper, we extend our analysis to other parts of the *Almagest* and study the dating problem for series of lunar eclipses described in the *Almagest* and for the covering of stars by planets. The results obtained completely agree with our previous results and give the same time interval, AD 600–1300.

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1. Introduction

This paper is a natural extension and continuation of the authors' investigations into the astronomical dating problem of Ptolemy's famous *Almagest*. In [1–4], the authors suggested and developed a new geometrical-statistical method for the dating of ancient star catalogues. This method was tested on several medieval catalogues whose dates are well known (Tycho Brahe, etc.) and also on several artificially generated (by computer) star catalogues. This experiment confirmed the efficiency of our method because it gave dates which agree with the real, well known dates of these catalogues. Then the same method was applied to the star catalogue from the *Almagest*. It turned out that the result obtained does not confirm the traditional dating of the *Almagest* (2nd century AD or 2nd century BC) but shifts its dating to the Arabian epoch AD

600–1300. This catalogue cannot be created (observed) outside this time interval. Moreover, the most possible date of its creation is the 10th century AD.

This evidently contradicts the standard date of the creation of the *Almagest*, presumably AD 137. Thus, a serious problem arises: 'Is the star catalogue from the *Almagest* 'a late insertion' into the original, ancient and authentic text, or was the whole text of the *Almagest* (or its major part) written later than AD 600 and finally edited in the late Middle Ages (AD 1200–1300)?'

The astronomical observations collected in the *Almagest* were recently studied by the well-known American scientist, astronomer, and specialist in celestial mechanics, navigation, and astrophysics, Robert R. Newton [5]. The result of his analysis can be briefly formulated as follows:

(1) The *Almagest* contains the theory of the Moon's motion, the Sun's motion, planets' motion, and precession theory.
(2) A large part of the astronomical data (many 'observations') collected in the *Almagest* can be theoretically calculated on the basis of Ptolemy's theory.
(3) The greater part of the astronomical 'observations' is indeed nothing more than the result of such 'pure theoretical calculations', which were made (according to Newton's results and opinion) by Ptolemy itself.

Consequently, it is senseless using these 'data' for an independent astronomical dating of the *Almagest*, because it implies only a reconstruction of the opinion (or the conjecture) of some later author (Ptolemy or some medieval astronomer?) about the dates of these astronomical events. Medieval authors sometimes tried to solve the following problem: in which month of what ancient year (epoch) did certain concrete astronomical events occur?

But, fortunately, the *Almagest* contains some astronomical observations which cannot be calculated only on the basis of Ptolemy's theory or on the basis of the latest medieval astronomical theories. Among them are the ecliptical *latitudes* of 1020 stars in the star catalogue of the *Almagest*. This sufficiently large number of real astronomical observations was used in our previous work for dating the star catalogue in the *Almagest* (this problem was successfully solved; see [1–4]).

It turned out that the *Almagest* also contains some other 'noncalculable' (in the Middle Ages) observation data. Among them are

(A) Four observations of the covering of stars by moving planets;
(B) Twenty-one observations of lunar eclipses mentioned in the *Almagest*.

Our present work is devoted to dating the *Almagest* on the basis of observation data A and B. Let us emphasize that here we actually date the *text of the Almagest itself* and not only its star catalogue, as in [1–4]).

We obtained the following results:

(1) Observation data A can be dated in the historical interval from AD 887 to AD 1009. It is remarkable that this time interval agrees with the interval which was obtained in [1–4] as a result of the independent star catalogue's dating.

(2) Observation data B are distributed, according to the *Almagest*, over a long time interval (about 900 years). It turned out that this is the historical interval from AD 492 to AD 1350. Moreover, the most 'dense' collection of observations of lunar eclipses occurred in the 11th century. And again we see the ideal correspondence with the results of the independent dating of the star catalogue of the *Almagest* and of observation data A (see above).

(3) In both cases A and B, Ptolemy assigned observations to the same 'era' (the so-called 'era of Nabonassar'). It is clear that, after dating all observations A and B, we can now obtain the *beginning* (*the initial point*) *for this era* by two *independent methods*. It is remarkable that these methods give rise to the same result: *the beginning of Nabonassar's era is about* AD 490. Let us recall that the traditional dating of this initial point (which is common today) is 747 BC.

It is important that the following numerical data:

the latitudes in the star catalogue of the *Almagest*,
the information about the coverings of the stars by planets, and
the observations of lunar eclipses in the *Almagest*,

are *completely independent*. Thus, an excellent coincidence of all these datings in all three cases, is a serious argument in favour of the opinion that the *Almagest* is the entire (righteous) document (text) which was originally created between the 10th and 11th centuries and was then extended and enlarged by the middle of the 14th century.

2. The Dating of the Covering of Stars by Planets

The *Almagest* contains a description of only four coverings of stars by the planets [5, 9]. Ptolemy says:

(1) Of the old observations, we took one which Timocharis records thus: In the year 13 of Philadelphus, Egyptianwise Mesore 17-18 at the twelfth hour, Venus appeared to have exactly overtaken the star opposite Vindemiatrix [7, p. 319; Section X.4].

(2) We took one of the old observations to which it is quite clear that in the year 13 according to Dionysius, Aigon 25 in the morning, Mars seemed to occult the Scorpion's northern forehead [7, p. 342; Section X.9].

(3) We again took one of the ancient observations very faithfully recorded, according to which it is quite clear that in the year 45 of Dionysius, Parthenon 10, Jupiter at sunrise occulted the Southern Ass [7, p. 361; Section XI.3].

(4) We took for this again one of the faithfully recorded ancient observations, according to which it is clear that in the year 82 of the Chaldeans, Xanthicus 5, in the evening, Saturn was 2 digits below the Virgin's southern shoulder [7, p. 379; Section XI.7].

According to traditional identifications of Ptolemy's stars compared to modern ones [5, 9], we now have information about the following coverings:

(1) Venus covered the star η Var at about midnight.
(2) Mars covered the star β Sco in the morning.
(3) Jupiter covered the star δ Cnc at sunrise.
(4) Saturn was '2 digits (2 units?)' below the star γ Vir.

We checked all these traditional identifications and they were confirmed. For the calculation of planet locations in the past, we used a modern theory and concrete values of the averaged elements of planet orbits from the well-known book by G. N. Duboshin [8], see Appendix 1. The accuracy of the calculations of the latitude position is equal to 1′ (1 minute).

Let us now comment how one needs to understand '*a planet covered the star*'.

It is well known that the normal human eye can distinguish two points at an angular distance of about 1′. Extremely strong eyes can distinguish two points at an angular distance of about 30″. Consequently, the covering ('coincidence') of the star by some planet, in reality means that the angular distance between them (from the point of view of the astronomer on the Earth's surface) is equal to about 1′. It is clear that it was impossible for Ptolemy to calculate (even in principle) this remarkable astronomical event, because of the accuracy of his theory was about 10′! Modern theory allows us to calculate the latitude positions of Venus and Mars in the past (on the historical time interval under the consideration) with an accuracy 1′. The accuracy of calculations of the longitudes of Mars and Venus is equal to about 3′. This is sufficient for us because only the value of the latitude actually determines the covering of the star by the planet. The longitude of the planet changes rapidly (in comparison with the latitude) and we can assume that the longitude is proportional to time. Consequently, a small error in the calculation of the longitude implies only a small error in the calculation of the covering time. Thus, in the cases of Mars and Venus, the covering described by Ptolemy can be calculated with great accuracy on the basis of modern theory.

The theory of the motion for Jupiter and Saturn is more complicated and less accurate than for the case of Mars and Venus. V. K. Abalakin writes:

The averaged elements of the orbits of Jupiter, Saturn, Uranus, Neptune, Pluto cannot be used for the solving of stability problem and cannot serve during the millions of years... They are suitable during several centuries from our epoch [6, p. 302].

But the situation in the *Almagest*'s case is such that we do not need exact formulas for Jupiter and Saturn. Really, according to the *Almagest*, the observation of Saturn has only an auxiliary meaning because Saturn did not cover the star but was at an uncertain distance of 'two units (digits)' from the star. What Ptolemy meant here by the term 'digit' (unit) is not quite clear. Consequently, it is senseless to calculate the position of Saturn with accuracy 1′.

In the case of Jupiter, Ptolemy states that "*Jupiter covered* (*occulted*) *the star*". But our computer calculations on the basis of modern theory shows that the angular distance between Jupiter and δ Cnc has never been less than 15′ (!) on the whole historical time interval. Consequently, we can only try to find such moments when the distance between Jupiter and the star δ Cnc is about 15′–20′. We do not need a high accuracy of the formulas for this purpose. The accuracy which is guaranteed by modern theory is sufficient.

Let us discuss the question of how Ptolemy distributes the astronomical events (1)–(4) over the time axis. The universal 'era' for Ptolemy is 'the Era of Nabonassar'.

Table I

Covering of the star by the planet	Year according to Ptolemy: Era of Nabonassar	Era 'after the death of Alexander'	Era of Dionysius
1. Venus	406		
2. Mars	476	42	13
3. Jupiter		83	45
4. Saturn	519		

Usually, he assigns dates in terms of this era to different astronomical events, though sometimes he uses other eras. Table I contains all datings of the coverings according to Ptolemy. One can see that Ptolemy used (at least twice) the following three eras: Nabonassar, 'after the Death of Alexander', and Dionysius.

Investigation of this table shows that Ptolemy's chronology contains some errors (disagreements). The time distance between the coverings of the stars by Mars and Jupiter is equal to 41 year if we use the era of Alexander. But the same distance is equal to 32 years if we use the era of Dionysius. This implies two versions in terms of the era of Nabonassar: 517 and 508 years. We consider both versions.

Thus, we can now state an exact mathematical problem. Namely, we must find the year N, launching the following chain of astronomical events:

(1) In the year N, Venus covered the star η Vir at about midnight.
(2) In the year $N + 70$, Mars covered the star δ Sco in the morning.
(3) In the year $N + 111$ (or $N + 102$), Jupiter covered the star δ Cnc at sunrise.
(4) In the year $N + 113$, Saturn was near the star γ Vir (below).

Let us discuss the accuracy which is needed for the time distances between the different coverings. It is clear that we need to take into account all possible errors because of Ptolemy's reduction of all dates to the same era (Nabonassar). It is evident that this recalculation can lead to errors of 1–2 years because different eras used different beginnings of the calendar year. It is well known that the beginning of the year was placed at the March, August, September, October, January, etc., in different eras (sometimes even the variable starting point of the year was used!). So, it would not be surprising to encounter errors of several years. The best solution we found has the error of 4 years.

ASSERTION 1. *Only two solutions exist that are mentioned in the time interval 500 BC–AD 1600.*

First Solution (medieval):

(1) At AD 887, September 9, at midnight Greenwich Mean Time, Venus covered η Vir (the calculated distance between them is less than $1'$).
(2) At AD 959, September 27, at 6 hours, 50 minutes Greenwich Mean Time, Mars covered β Sco (the calculated distance is equal to $3'$).

(3) At AD 994, August 13, at 5 hours 15 minutes Greenwich Mean Time, the distance between Jupiter and δ Cnc was about 20′. This distance is close to the absolute minimum of the possible distance between Jupiter and δ Cnc in the time interval under the consideration.
(4) At AD 1009, September 30, at 4 hours 50 minutes Greenwich Mean Time, Saturn was at a distance equal to 50′ from the γ Vaiir (below the star).

Second Solution (ancient):
(1) At 328 BC, September 1, at 21 hour 30 minutes Greenwich Mean Time, Venus covered η Vir (the calculated distance is less than 1′).
(2) At 256 BC, September 17, at 5 hours 10 minutes Greenwich Mean Time, Mars covered β Sco (the calculated distance is less than 1′).
(3) At 228 BC, September 9, at 4 hours 15 minutes Greenwich Mean Time, Jupiter was at a distance of about 15′ from δ Cnc. This distance is close to the absolute minimum for the distance between Jupiter and this star on the whole historical time interval.
(4) At 228 BC, September 6, at 15 hours 10 minutes Greenwich Mean Time, Saturn was at a distance equal to 127′ from γ Vir (below the star).

For both solutions, the errors for the time intervals between the successive observations (events), respectively, compared to Ptolemy's time intervals is less than or equal to 4 years. If we delete Saturn, then for the first (medieval) solution, we obtain only 3 years as the time error. To obtain other (additional) solutions, we must enlarge the time error up to 10 years. (This is the statement about the stability of our result.)

All dates in Assertion 1 are given in terms of the Julian calendar with the beginning of the year at January 1.

The 'solution' of this problem which is usually suggested by the chronologists of 16th–18th centuries (Newton [5]) is as follows:

(1) 272 BC (−271), October 12. Venus 'touched' η Vir, but the distance between Venus and the star does not exceed 15′ (!).
(2) 271 BC, January 18 (or 16). Mars 'touched' β Sco. But actually the distance between Mars and the star was about 50′ at January 18, and about 15′ at January 16 (!).
(3) 241 BC, September 4, Jupiter 'covered' δ Cnc. But the calculation shows that the distance between Jupiter and the star at this moment was more than 25′.
(4) 229 BC, March 1. Saturn was at a distance of "2 units' (digits) from γ Vir. But, as we have discussed, the authenticity of this observation depends on the meaning of the term 'digit'.

It is quite clear that this cannot be considered as a solution to the problem. We must state that the chronologists who studied the *Almagest*, did not satisfy Ptolemy's conditions. Besides, they based their 'solution', not on the correspondence between the data given by Ptolemy and modern calculations, and not even on the time distances between successive observations also given by Ptolemy, but on the doubtful interpretation of the names of the months given by Ptolemy. They also based their 'solution' on astronomical characteristics (the longitude of the Sun, the time of the observation, the longitude of the planet, etc.) *calculated by Ptolemy* with the help of his approximate theory (he wrote that he *calculated* these characteristics). Consequently, all these latest Ptolemy calculations were added by him to the ancient information about these coverings. Of course, such calculations cannot be used for independent dating of ancient observations. Besides, as we have seen from our analysis, the chronologists have totally ignored the ancient data which were *quoted* by Ptolemy and which he did not calculate. These data are: the year of the covering and the fact of the covering itself.

Let us note that first (medieval) solution ideally agrees with the independent dating of the star catalogue of the *Almagest* [1–4]. Let us recall that this dating was obtained on the basis of a very detailed and stable statistical analysis of the whole star catalogue. If we consider the *Almagest* as the entire scientific text (as historians do), we must consider only the first (medieval) solution as the real one. But it would be dishonest to hide the second (ancient) solution which is at a distance about 1200 years from the first one and its existence can lead to further hypotheses. Note that this solution does not coincide with the traditional one. Its appearance can be explained by different reasons, e.g. by a periodicity in the effect of the covering of stars by planets. Namely, the plane configuration of Earth and the planets changes with time in accordance with approximately periodic law. This configuration determines such astronomical events as the covering of stars by planets (which are visible from the Earth). Thus, it is quite natural that we have found two solutions to our problem (Figure 1).

COROLLARY. *The first solution of the dating problem (see Assertion* 1) *implies that the beginning of the Era of Nabonassar (in the chronology of the* Almagest) *must be settled at* AD 489—490.

3. The Dating of the Lunar Eclipses

The 21 lunar eclipses mentioned in the *Almagest* were observed by different astronomers approximately during the time interval from 26 till 881 years of Nabonassar. Ptolemy listed the following characteristics of the eclipses:

(1) The year of the eclipse in terms of some chronological era, which was given in the ancient document used by Ptolemy. Usually, after this, Ptolemy recalculated this year in the era of Nabonassar. In several of the remaining cases, this recalculation can be easily done on the basis of the relations between the different eras which are listed in the *Almagest*.
(2) The phase of the eclipse according to the ancient document which is quoted by Ptolemy. Let us recall that the *Almagest* contains the theory of the Moon's motion. But this theory did not allow Ptolemy to calculate the phase of the

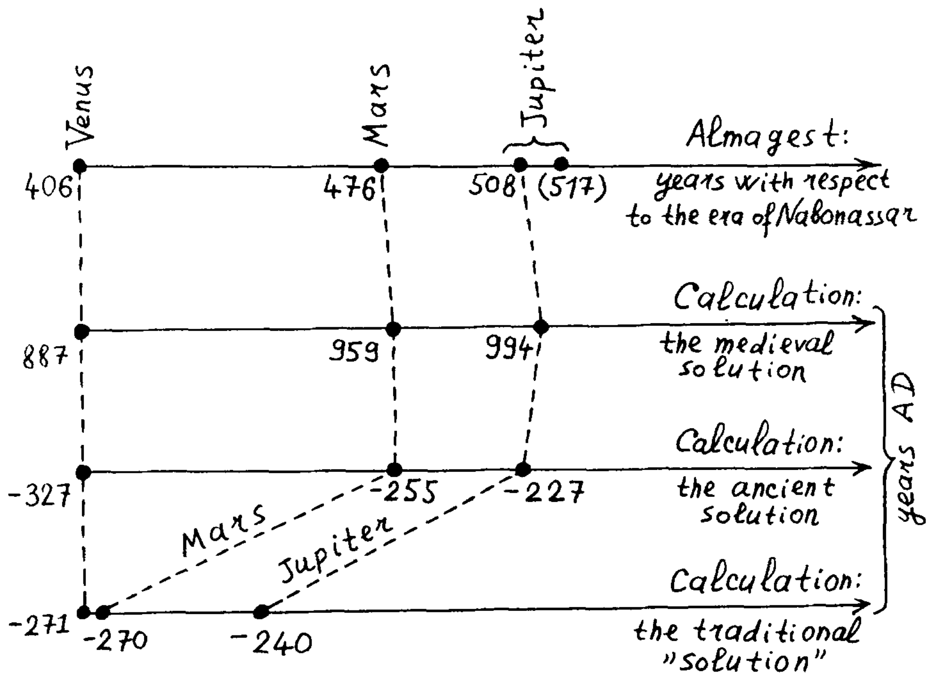

Fig. 1.

eclipse. This is the reason why he quoted the phase from the ancient text without any comments. A more advanced theory of the Moon's motion allowing the calculation of the phase of lunar eclipses was created only during the 19th century AD.

(3) The date of the eclipse and the time of the 'middle of the eclipse'. These data are the result of Ptolemy's calculations. Consequently, these 'calculated data' are not of any interest for an independent dating problem.
(4) The place of the observation of the eclipse. Note that any lunar eclipse is visible from half of the Earth's globe. Hence, the indication of the place is not of serious significance.

Thus, only data (1) and (2) are really important for the dating problem, because Ptolemy did not calculate these data and simply extracted from from ancient documents.

It follows, that we use

(1) The year of the eclipse in terms of some chronological era (its beginning we assume to be unknown but we calculate it after solving the dating problem);
(2) the phase of the eclipse.

Let us recall that the phase of an eclipse is equal to the maximal part of the diameter of the Moon which is shadowed; this part is measured by units which are equal to 1/12 of this diameter. For supertotal eclipses, we need to calculate the length of the Earth's

Table II.

No. eclipse	Era of Nabonassar (year)	Hour of the middle of the eclipse in Alexandria.[a]	Phase of the eclipse (standard units)
1	26	21	total
2	27	23	3
3	27	20	6
4	127	5	3
5	225	22	6
6	246	24	3
7	256	23	2
8	366	6	1
9	367	23	total
10	546	19	9
11	547	1	total
12	547	2	total
13	574	2	7
14	607	22	3
15	870	20	2
16	878	23	total
17	880	22	10
18	881	4	6

[a]Calculated by Ptolemy.

shadow which is crossed by the Moon. The total eclipse starts from 12 units (all eclipses with phases more than 12, are total). Ptolemy does not mention the phase for three eclipses from the 21 mentioned in the *Almagest*. But at each point of the Earth's surface, one can observe at least one lunar eclipse a year (with some phase). Consequently, mentioning these eclipses without their phases does not carry any real astronomical information. Thus, we are forced to exclude these three eclipses and work with the remaining 18 which are listed in Table II.

The problem of independent astronomical dating of the lunar eclipses in the *Almagest* can be stated as follows. From the past, we need to find (on the basis of the modern theory of the Moon's motion) the set of 18 lunar eclipses which satisfy the following conditions.

(1) Each eclipse must have the phase which is given in the *Almagest* (within an accuracy of 1 unit). The phases of the eclipses were determined by medieval astronomers sufficiently accurately (from visual observation) and have not since been changed by recalculations. Thus, we can assume that the phase of the lunar eclipses in the *Almagest* is correctly quoted within an accuracy of 1 unit (because the value of the phase is represented in the *Almagest* by an integer number of units).

(2) The 'inter-eclipses times' must correspond to the distances which are listed in the *Almagest*. But because Ptolemy used several different ancient documents, the years of some eclipses are given, respectively, to different eras. It is impossible to demand an accuracy of better than 2 years between the eclipses. The reason is (see our discussion above) that the different eras can fix different beginnings of the year. Consequently,

recalculation from one era to another can produce a natural error equal to 1 year. For the difference between two dates, this error, consequently, can be equal, to 2 years.

We have solved this numerical problem with the help of computer calculations and the modern theory of the Moon's motion. We have also tested our results by comparing them with the well-known Canons of the eclipses [10–11]. We considered *all* eclipses of the historical interval from 900 BC to AD 1600. The result obtained is the following assertion.

ASSERTION 2. *A unique solution exists to the problem of the dating of lunar eclipses in the* Almagest *which satisfies, within an accuracy of* 3 *years, all conditions imposed on inter-eclipses times and having the necessary phases. This is the set of eclipses collected in Table III. It turns out that all these eclipses are medieval.*

This unique solution is stable respectively for variations of time. Ptolemy used different ancient documents describing the lunar eclipses. These documents sometimes use different chronological eras. For example

- Eclipse Nos. 1–3 are dated in the ancient documents (as Ptolemy says) in the era of Mardokempad;
- eclipse Nos. 4–5 – in the era of Nabonassar;
- eclipse Nos. 6–7 – in the era of Darius;

Table III.

Eclipse No.	Date of eclipse				Phase of eclipse	Coordinates of the zenith point of the eclipse on the Earth	
	Year AD	Day	Month	Hour (Greenwich)		Longitude	Latitude
1	491	5	8	16	11.1	110	−17
	or						
	492	30	1	16	16.7	123	17
2	494	5	6	1	2.0	−28	−22
3	496	6	11	21	5.0	27	17
4	594	6	8	23	4.0	16	−17
5	693	27	3	14	5.6	138	−4
6	717	28	6	13	3.0	155	−23
7	728	27	5	21	2.5	31	−22
8	840	20	5	5	1.4	−77	−21
9	843	19	3	19	14.1	73	−1
10	1019	16	9	23	9.4	10	−1
11	1020	12	3	7	18.1	−111	1
12	1020	4	9	23	18.7	13	6
13	1046	23	4	7	6.6	−116	−14
14	1079	20	1	3	4.0	−48	19
15	1344	23	9	1	2.4	−31	3
16	1349	30	6	23	21.7	1	−23
17	1349	25	12	12	9.8	178	23
18	1350	20	6	17	5.8	103	−23

- eclipse Nos. 8–9 – in the Athenian magistracy;
- eclipse Nos. 10–12 – in the 3rd Callippic Period;
- eclipse No. 13 is assigned to the era of Philometor;
- eclipse Nos. 15–18 – in the era of Hadrian.

As we have seen, when recalculating from one era to another, Ptolemy made some errors (sometimes about 10 years). This means that, generally speaking, he does not give the exact position of the initial points for different eras. Consequently, the time distances between the eclipses which are given in terms of the same era, must be considered as more reliable in comparison with the distances between the eclipses assigned to different eras. The reason is that, in the first case, Ptolemy simply extracted the time differences from some ancient document and, consequently, these values do not depend on the position of the eclipses in an absolute time scale. But, in the second case, the time distances depend on Ptolemy's recalculations of dates belonging to different ancient eras to those belonging to the 'era of Nabonassar'. These recalculations can also produce additional errors.

This is the reason why we decided to continue our computer calculations to study the problem 'Are there any other solutions to our problem if we permit possible errors in time distances to increase?'. We decided to leave an accuracy of 3 years for inter-eclipse times belonging to the same chronological era, and to permit the accuracy to increase by up to 30 years (!) for inter-eclipse times 'connecting" eclipses assigned to different eras.

Remark. The eclipses assigned (in the *Almagest*) to the same era, form compact groups on the time axis, i.e. they are located inside sufficiently small time intervals. But distances between successive eclipses, assigned to different eras, are some tens and hundreds of years. In other words, the eclipses form some condensations on the time axis. It is clear that each such condensation is a reflection of some homogeneous set of observations which were made (according to the *Almagest*) by the same scientific school, may be, more or less, in the same place. Consequently, it is natural to think that the mutual position of the eclipses inside each (homogeneous group' must be more precise than the mutual position (on the time axis) of the condensations. The location of these condensations on the common time scale is evidently the result of later chronological work and recalculations.

ASSERTION 3. *Let us consider the accuracy of* 3 *years for inter-eclipse times for successive eclipses assigned to the same era, and an accuracy of* 30 *years for inter-eclipse times for successive eclipses assigned to different eras. Then the solution found in Assertion* 2 *still remains unique on the whole historical time interval under consideration.*

If we enlarge the accuracy up to 4 years for all cases, then a new solution appears with the first eclipse at 721 BC. This solution is close to the traditional one suggested by historians and chronologists, but does not coincide in detail with traditional datings. Figure 2 shows two histograms which demonstrate the distribution of deviation (in comparison with the *Almagest*) of inter-eclipse times for both solutions.

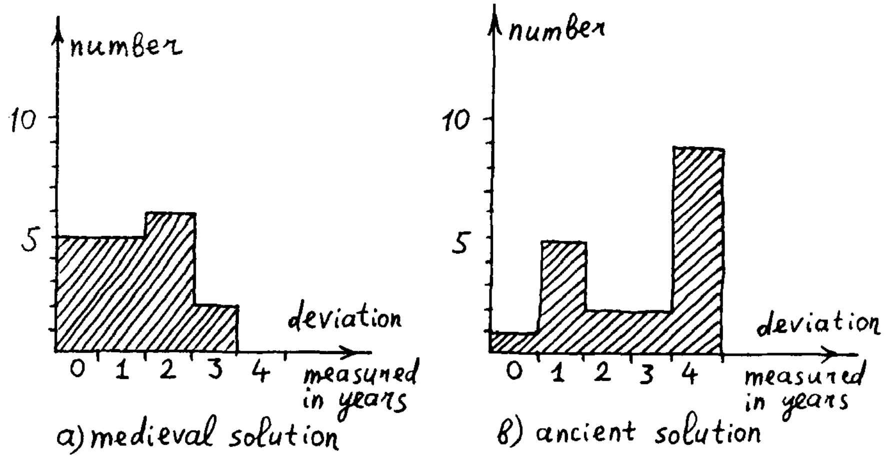

Fig. 2.

It is clear that the first (medieval) solution is considerably better than the second one (ancient).

Here (as in the case of star coverings), we also have the periodicity in the lunar eclipses. The existence of the second (ancient) solution is explained by the approximate periodicity in the evolution of the configuration consisting of Sun, Earth, Moon. This period is of about several hundreds years. But the periodicity has only an approximate character and it follows that the second (ancient) solution is considerably poorer than the first (medieval) one.

4. Mathematical Chronology of the *Almagest* (Figure 3)

According to our dating of the star coverings by the planets the era of Nabonassar in the *Almagest* starts at AD 470–490. More precisely, the exact dates for this starting point, obtained on the basis of different star coverings, and on the basis of different versions connected with an 11-year disagreement in the internal chronology of the *Almagest*, are as follows: AD 477, AD 481, AD 483, AD 486.

The dating on the basis of the collection of lunar eclipses in the *Almagest* gives AD 465 as the first year of Nabonassar. What can we say about the accuracy of this value? A comparison of the time configuration of the eclipses in the *Almagest* with the real time configuration discussed above, shows that the global chronology of the *Almagest* contains some errors (displacements) which have the same value as for the case of star coverings (the maximal chronological displacement is equal to 11 years). Consequently, a typical accuracy of the relative positions of the basic points for different eras (their initial points) in the *Almagest* is 10–15 years.

The agreement between our datings resulting from star coverings and lunar eclipses is ideal. They both lead to the same interval, AD 460–490, which is supposed to contain the beginning of the era of Nabonassar.

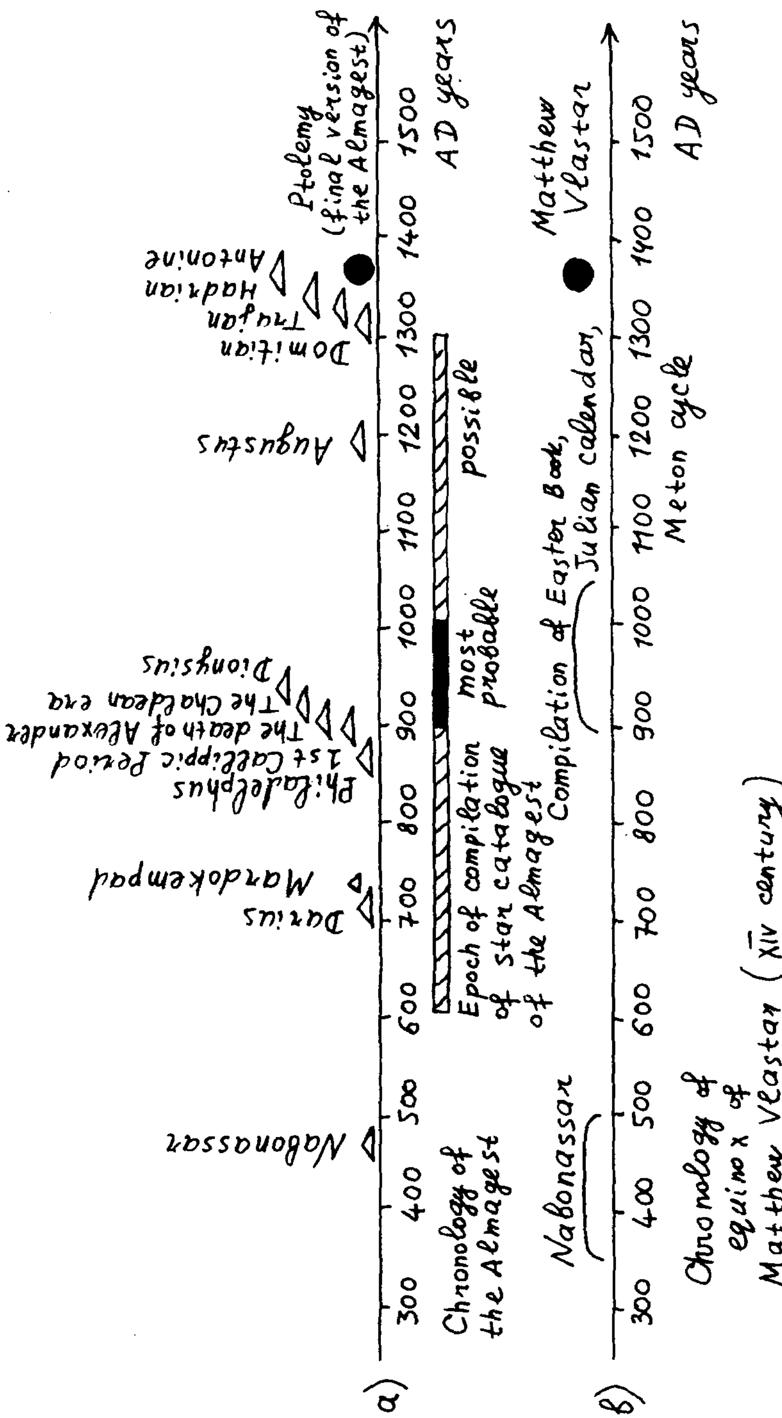
a)
Nabonassar
Darius
Mardokempad
Philadelphus
1st Callippic Period
The death of Alexander
The Chaldean era
Dionysius
Augustus
Domitian
Trajan
Hadrian
Antonine
Ptolemy (final version of the Almagest)
300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500
AD years
Chronology of the Almagest
Epoch of compilation of star catalogue of the Almagest
most probable
possible
b)
Nabonassar
Compilation of Easter Book, Julian calendar,
Matthew Vlastar
300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500
AD years
Meton cycle
Chronology of equinox of Matthew Vlastar (XIV century)

Fig. 3.

Now we can reconstruct the global chronology of the *Almagest*. In the *Almagest*, Ptolemy mentions the dates (in terms of the era of Nabonassar) of the following events from the history of Assyria, Egypt, and Rome:

(1) the rule of Darius,
(2) the rule of Philadelphus,
(3) the beginning of the Callippic periods,
(4) the death of Alexander (it is usually assumed that here Ptolemy means Alexander of Macedonia, but really Ptolemy simply mentions some 'Alexander'),
(5) the beginning of the Chaldean era,
(6) the beginning of the era of Dionysius,
(7) the rule of Augustus (book III.7),
(8) the rule of Domitian (book VII.7).
(9) the rule of Trajan (book VII.7),
(10) the rule of Hadrian,
(11) the rule of Antonine.

For all these events, we automatically obtain the following dates (the time intervals are considered within an accuracy of 5 years):

(0) the beginning of the era of Nabonassar: AD 460–490,
(1) the rule of Darius: AD 685–715,
(2) the rule of Philadelphus: AD 840–885,
(3) the beginning of Callippic Periods: AD 875–910,
(4) the death of Alexander: AD 885–915,
(5) the beginning of the Chaldean era: AD 900–935,
(6) the beginning of the era of Dionysius: AD 915–945,
(7) the rule of Augustus: AD 1175–1205,
(8) the rule of Domitian: AD 1290–1320,
(9) the rule of Trajan: AD 1310–1340,
(10) the rule of Hadrian: AD 1310–1345,
(11) the rule of Antonine: AD 1330–1365.

5. Summary

(1) The reconstructed chronology of the *Almagest* ideally corresponds to the dating of the star catalogue of the *Almagest*: AD 600–1300 (where the most plausible time interval of the creation of the catalogue is the 10th century AD). According to this chronology the following events (mentioned in the *Almagest*) book place between the 9th and 10th centuries AD:

- all observations of star coverings by the planets;
- the most massive condensations of the observations of the lunar eclipses;
- initial points of the most important chronological eras such as the era of

Philadelphus, Callippic periods, the era of Alexander, Chaldean era, the era of Dionysius – a total of 5 eras from the 11 mentioned in the *Almagest.*

(2) The time interval for the death of Alexander (AD 885–915, according to the reconstructed chronology of the *Almagest*) practically coincides with the rule of the *unique* emperor Alexander AD 912–913 in the history of Byzantium and Western Europe.

(3) The time interval for the beginning of the Callippic periods covers the starting point of the Great Indiction at AD 877. Let us recall that the starting points of the Great Indictions are at a distance of 532 years from each other. This is the time period after which combinations of medieval calendar characteristics of the year (such as indict, Moon's cycle, Sun's cycle) are repeated. But the shorter period was also used for cycles. This is the so-called Callippic period (Cycle) which is equal to 76 years. One Great Indiction consists of an integer number of Callippic periods. Consequently, it is natural to expect that the Callippic period is simply a subdivision of the Great Indiction and, hence, the beginning of the Great Indiction must coincide with the beginning of the 1st Callippic period. It turns out that this natural conjecture is completely confirmed in the reconstructed chronology of the *Almagest*: the 1st Callippic period starts at AD 877 – exactly at the year which is the beginning of the Great Indiction.

Appendix 1. Formulas for Planet Positions

We used the following formulas for the calculation of the positions of the planets and the Earth in the ecliptical coordinates. In these formulas T denotes the time which is measured in Julian centuries (i.e. 36525 ephemerid days) from the epoch 1900, January 12^{h}ET. Here ET = ephemerid time.

In these formulas L is the mean longitude of the planet at the moment T, π is the mean longitude of the perigee of the orbit of the planet, e is the eccentricity of the orbit, Ω is the longitude of the ascending knot of the orbit, and a is the main half-axis of the elliptic orbit.

The parameters L, π, and Ω refer to the instantaneous ecliptic at the moment T.

Earth

$L = 99°41'48''.04 + 129602768''.13T + 1''.089T^2,$

$\pi = 101°13'15''.0 + 6189''.03T + 1''.63T^2 + 0''.012T^3,$

$e = 0.01675104 - 0.0004180T - 0.000000126T^2,$

$i = 0,$

$a = 1.00000023.$

Mars

$L = 293°44'51''.46 + 68910103''.83T + 1''.1184T^2,$

$\pi = 334°13'05''.53 + 6626''.73T + 0''.4675T^2 - 0''.0043T^3,$

$\Omega = 48°47'11''.19 + 2775''.57T - 0''.005T^2 - 0''.0192T^3,$

$e = 0.09331290 + 0.000092064T - 0.000000077T^2,$
$i = 1°51'01''.20 - 2°.430T + 0''0454T^2,$
$a = 1.52368840.$

Venus

$L = 342°46'01''.39 + 210669162''.88T + 1''.1148T^2,$
$\pi = 130°09'49''.8 + 5068''.99T - 3''.515T^2,$
$\Omega = 75°46'46''.73 + 3239''.46T + 1''.476T^2,$
$i = 3°23'37''.07 + 3''.621T - 0''.0035T^2,$
$e = 0.00682069 - 0.00004774T + 0.000000091T^2,$
$a = 0.72333162.$

Jupiter

$L = 238°02'57''.32 + 10930687.148T + 1''.20486T^2 - 0.005936T^2,$
$\pi = 12°43'15''.34 + 5795''.862T + 3.80258T^2 - 0''.91236T^3,$
$\Omega = 99°26'36''.19 + 3637''.908T + 1''.2680T^2 - 0''.03064T^3,$
$e = 0.04833475 + 0.000164180T - 0.0000004676T^2 - 0.0000000017T^3,$
$i = 1°18'31''.45 - 20''.506T + 0''.014T^2,$
$a = 5.202561.$

Saturn

$L = 266°33'51''.76 + 4404635''.5810T + 1''.16835T^2 - 0''.021T^3,$
$\pi = 91°05''53''.38 + 7050''.297T + 2''.9749T^2 + 0''.0166T^3,$
$\Omega = 112°47'25''.40 + 3143''.5025T - 0''.54785T^2 - 0''.0191T^3,$
$e = 0.05589232 - 0.00034550T - 0.000000728T^2 + 0.00000000074T^3,$
$i = 2°29'33''.07 - 14''.108T - 0''.05576T^2 + 0''.00016T^3,$
$a = 9.554747.$

These parameters and equations determine the position of the Earth and the planets in the space at every moment T. We calculated the direction Earth-planet and compared this direction with the star position on the celestial sphere. The covering moment is exactly the time when these two directions (Earth-star and Earth-planet) coincide. We have investigated the stability of the mentioned direction's calculation. It turned out that this calculation has an accuracy of about 1′ in latitude. This is quite enough for our purposes.

Appendix 2. The Table of the *Almagest*'s Lunar Eclipses

(1) "Then of the three ancient eclipses observed in Babylon, of which we spoke, the first is recorded as having taken place in the year 1 of Mardokempad... And the eclipse began, it is stated, more than one hour after the rise of the Moon, and the eclipse was total" [7, p. 123; book IV.6].

(2) "The second of the eclipses is recorded as having occurred in the year 2 of Mardokempad... And there was an eclipse, it says, of 3 digits from the southern end

at midnight" [7, p. 123, book IV.6]. The comment of R. C. Taliaferro: "A digit is 1/12 of the moon's diameter".

(3) "The third of the eclipses is recorded as having taken place in the same year 2 of Mardokempad... And the eclipse began, it says, after the rise of the Moon, and there was an eclipse of more than half from the northern end" [7, p. 123, book IV.6].

(4) "For the year 5 of Nabopollassar (which is the year 127 of Nabonassar...) the Moon began to be eclipsed in Babylon; and the greatest extent of the eclipse was 1/4 of the diameter from the south" [7, p. 172, book V.14].

(5) "Again, in the year 7 of Cambyses (which is the year 225 of Nabonassar...) the Moon was eclipsed in Babylon to the extent of a half of its diameter from the north" [7, p. 172, book V.14].

(6) "The second is the Hipparchus used, occurring in the year 20 of Darius, successor of Cambyses... And here likewise the Moon was eclipsed to the extent of a quarter of its diameter from the southern side" [7, p. 137, book IV.9].

(7) "We then book, first, the eclipse observed in Babylon in the year 31 of Darius... and the Moon was eclipsed to a breadth of 2 digits from the southern side" [7, p. 136, book IV.9].

(8) "Now he says there three eclipses were given out by those crossing over from Babylon as having been observed there, that the first of them occurred in the Athenian magistracy of Phanostratus... the Moon was eclipsed to the extent of a small bit of its circle... And he says it was still eclipsed when setting" [7, p. 140, book IV.11].

(9) "Again, he says, the next eclipse occurred in the Athenian magistracy of Phanostratus... And the Moon was eclipsed... This date is the year 366 of Nabonassar" [7, p. 140, book IV.11]. Here – no phase.

(10) "And he says the third eclipse occurred in the Athenian magistracy of Evandrus... And the eclipse, he says, was total... This date is the year 367 of Nabonassar" [7, p. 141, book IV.11].

(11) "And next we shall pass to the three later eclipses set out by him, which he says were observed in Alexandria. He says the first of these occurred in the year 54 of the Second Callippic Period... the Moon began to be eclipsed 1/2 hour before rising and returned to its full size in the middle of the third hour" [7, p. 141, book IV.11].

(12) "The next eclipse occurred, he says, in the year 55 of the same period... and the eclipse was total" [7, p. 142, book IV.11].

(13) "And he says the third eclipse occurred in the same year 55 of this Second Period... and the eclipse was total" [7, p. 142, book IV.11].

(14) "In the year 7 of Philometor then, (i.e., the year 574 of Nabonassar...), in Alexandria the Moon was eclipsed up to 7 digits from the north" [7, p. 196, book VI.4].

(15) "For example, from the observation of the eclipse in the year 32 of the Third Callippic Period...". [7, p. 80, book III.1]. Here – no phase.

(16) "Once again, in the year 37 of the Third Callippic Period (which is the year 607 of Nabonassar...), the Moon began to be eclipsed... and was obscured at the most 3 digits from the south" [7, p. 196, book VI.4].

(17) "... From the eclipse in the year 43 of the same period..." [7, p. 80, book III.1]. Here – no phase.

(18) "Second, we took that observed in Alexandria in the year 9 of Hadrian... and the Moon was eclipsed likewise to the extent of 1/6 of its diameter from the southern side" [7, p. 136, book IV.9].

(19) "Again, of the three eclipses we have chosen from those most carefully observed by us in Alexandria, the first occurred in the year 17 of Hadrian... And the eclipse was total" [7, p. 129, book IV.6].

(20) "The second occurred in the year 19 of Hadrian... And there was an eclipse to the extent of 1/2 + 1/3 of the diameter from the northern side" [7, p. 129, book IV.6].

(21) "The third of the eclipses occurred in the year 20 of Hadrian... And there was an eclipse to the extent of 1/2 of the diameter from the northern side" [7, p. 129, book IV.6].

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