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Mathematical Statistics and Problems of Ancient Chronology. A New Approach

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Abstract. New methods of processing statistically quantitative textual information of a narrative character are introduced. The methods can be used to recognize dependent and independent texts among large collections of texts. The methods are applied to the problem of correct dating of the events in ancient chronology. These results induce conjecture on the redating of some historical events.

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1. Introduction. Problems of the Correct Dating of Ancient Events

The previous paper in this issue (A. T. Fomenko, V. V. Kalashnikov, and G. V. Nosovsky, 'When was Ptolemy's star catalogue in 'Almagest' compiled in reality?) shows that there are serious problems concerning the correct dating of ancient events. We saw that the date of the compiling of the star catalogue in the Almagest belongs to the time interval AD 600–1300. This fact contradicts the standard date of the creation of the Almagest: 2nd century AD. How is it possible? How can we reconcile a strong statistical result with the traditional chronology of the history of science? The difference between the traditional date and our date is no less than 400 years.

Consequently, modern statistics gives us dates which are quite impossible from the point of view of standard traditional chronology.

The example of the Almagest is not unique. Here we will try to demonstrate to the reader an idea of the real extent of the problem.

Chronology is what tells us how much time has elapsed between some historical event and the present. To determine the real chronology, one must be able to translate the data in ancient documents into the terminology and units of modern time reckoning – i.e., into some date BC or AD. Many historical conclusions and interpretations depend upon what dates we ascribe to the events in a given ancient document. This problem is very complicated. Let us illustrate this with some examples.

Most modern dating methods are based on the comparison of data in an ancient document with events whose dates have been established earlier, and are regarded as known and fixed. E. Bickerman: "All the remaining dates in ancient chronology can be related to our calendar system by means of direct or indirect synchronizations with the Roman ones" ([1], p. 77). In other words, Roman chronology and history is the 'spinal column' of the entire present-day global chronology and history.

The chronology of ancient and medieval history in its present form was created and completed to a considerable extent in a series of fundamental works during the 16th and 19th centuries, beginning with J. Scaliger (1540–1609), the 'founder of modern chronological science' ([1], p. 88) and D. Petavius (1583–1652). However, the series of these (and other) works is not entirely complete. Indeed, as the well-known chronologist E. Bickerman observes, "there is no adequate, full-scale treatment of ancient chronology" ([1], p. 96, Note 1).

The absence of a modern study which would provide a strict scientific substantiation and construction of the global chronology of ancient times and the Middle Ages, based on modern data and methods, may be explained, not only by an enormous amount of historical material still requiring further processing and revision, but also by objective difficulties mentioned by various authors investigating at different times the scientific substantiation of chronology.

Consider, in particular, Roman chronology because of its leading role in the global chronology of ancient times. Extensive criticism of 'tradition' and its chronology began as far back as the 18th century at the *Academie Royale des Inscriptions et Médailles* founded in 1701 in Paris. In the 1720s, the general authenticity of Roman historical tradition was called into question (N. Fréret, L. de Pouilly). The accumulated material served as a basis for a more thorough criticism in the 19th century. The well-known historian T. Mommsen was one of the leading authorities in this branch of chronology, which was then called 'hypercriticism'. He wrote, e.g., that

> King Tarquinius the Second, although he was already grown up at the time of his father's death and did not begin to reign till thirty-nine years afterwards, is nevertheless still a young man when he ascends the throne. Pythagoras, who came to Italy about a generation before the expulsion of the kings [509 BC] is nevertheless set down by the Roman historians a friend of the wise Numa [died c. 673 B.C.; the discrepancy reaches at least 100 years]. The state-envoys sent to Syracuse in the year 262 transact busyness with Dionisius the Elder, who ascended the throne eighty-six years afterwards (348) ([2]d, V. III, p. 190)

The traditional chronology of Rome rests on a quite shaky basis, e.g., between different versions of the dating of such important events as the foundation of Rome, there exists a divergence of 500 years (*ibid.*). This oscillation says much about the dating of a large number of sources based on counting years from the 'foundation of Rome (City)' (e.g., the famous *History of Rome* by Livy). In general, "Roman traditional history has survived in the works of quite a few authors; undoubtedly, the most fundamental of them in Livy's" ([3], p. 3).

T. Mommsen:

. . . the prospect should be still more lamentable in the field . . . of the . . . annals . . . of the world . . . The increasing activity of antiquarian research induced the expectation that the curent narrative would be rectified from documents and other trust-worthy sources; but this hope was not fulfilled. The more and the deeper men investigated, the more clearly it became apparent what a task it was to write a critical history of Rome ([2], V. 5, p. 495).

Moreover:

falsification of numbers was here [regarding Valerias Antias] carried out down even to contemporary history . . . He [Alexander Polyhistor] . . . took the first steps towards filling up the five hundred years, which were wanting to bring the destruction of Troy and the origin of Rome into the chronological connection [as a matter of fact, according to another version of chronology, different from today's, Troy had fallen immediately before the foundation of Rome, and not 500 years before it] . . . with one of those lists of kings without achievements which are unhappily familiar to the Egyptian and Greek chronicles; for, to all appearance, it was he that launched into the world the kings Aventius and Tiberinus and the Alban *gens* of the Silvii whom the following times accordingly did not neglect to furnish in detail with name, period of reigning, and, for the sake of greater definitiveness, also a portrait' (*ibid.*, pp. 496–497).

A survey of this criticism can be also found in B. Niese [4] and a lengthy account of the ultrasceptical standpoint questioning the correctness of the 'regal Roman' chronology and, in general, the validity of our knowledge regarding the first five centuries (!) of Roman history can be found in [5] and [6].

As a matter of fact, Roman annals thenmselves do not survive; therefore, all our assumptions should be based on the Roman annalist historians. However, here, too, we face great difficulties, the main one being that the annalists' works are also in unsatisfactory state" ([3], p. 23).

The essential discrepancy found between the chronological data of ancient sources and the global chronology of ancient times, has been discovered in other of its branches, as well. Thus, considerable difficulties accompanied the establishment of Egyptian chronology, where a number of documents contradict each other chronologically, e.g., while accounting for the history of Egypt in a consecutive and connected manner, Herodotus places the Pharoahs Rhampsinitus and Cheops next to each other by calling the former the latter's successor. As a modern commentary states, "Herodutus mixes up the chronology of Egypt: Rhampsinitus (Ramses II) was a Pharoah of the XIXth dynasty (1345–1200 BC), while Cheops of the IVth in 2600–2480 BC" (see the Russian edition of Herodotus' *Histories*. Nauka, Leningrad, 1972, p. 513). Here, the discrepancy with today's chronological version amounts to 1200 years. It should be noted that, in general, Herodotus' chronology is essentially 'shorter' than modern chronology.

The famous Egyptologist H. Brugsch wrote that

German Egyptologists have attempted to fix the era when Mena, the first Pharoah, mounted the throne . . . The difference between the extreme dates is enormous, amounting to no less than 2097 years! . . .' ([23], p. 14).

Then Brugsch wrote, that the most authoritative works and researches carried out by competent scientists in order to check the chronological succession of the Pharoahs and whole dynasties proved, meanwhile, the inevitable necessity to suggest simultaneous and parallel reigns in Manetho's list, which considerably decreases the time required for the ownership of the country by thirty Manetho's dynasties. In spite of all discoveries in this branch of Egyptology, the numerical data were still in a quite unsatisfactory state at the end of the 19th century (*ibid.*).

It is not surprising that certain sceptical minds have made dramatic conclusions from the difficulties mentioned above. Thus, as early as the 16th century AD, a Professor of Salamanca University, de Arcilla, published two papers in which he stated that the whole of history earlier than the 4th century AD had been falsified (see de Arcilla, *Programma Historicae Universalis*, *Divinae Florae Historicae*). Similar conclusions were reached by the historian and archaeologist J. Hardouin (1646–1724) who considered a considerable part of classical literature as produced by the 16th century monks.

Isaac Newton devoted many years to historical and chronological studies. Having thoroughly investigated practically all the historical and theological literature, he wrote *Abrégés de la Chronologie* [7, 8], asserting that the chronology of ancient times was unnaturally extended. Newton made up his own tables in accordance with a new chronology which related the biblical texts with the history of the Mediterranean. V. G. Kuznetzov, in his book *Newton*, wrote that Newton had collected

> fantastically large volumes of historical material. This was the total of forty years of work, toilsome research and enormous erudition. Newton, in fact, studied all basis literature in ancient history and all primary sources . . . ([9], pp. 104–105).
>
> Certainly, being unable to read cuneiform and hieroglyphic texts, and having no archaeological data which were then unavailable, . . . Newton was in error to the extent not just of tens or hundreds of years, but even millennia . . ." (*ibid.*, pp. 106–107).

As a matter of fact, many of the important events of Greek history were chronologically moved by Newton forwards through 300 years, and those of Egyptian history through 1000, and even 1800 years. These events were 'shifted upwards' by I. Newton. The enormous work performed by Newton was sceptically received by his contempoaries. W. Whiston wrote: "Sir Isaac Newton composed a Chronology, and wrote 18 copies of its first and principal chapter with his own hand . . . which proved no better than a sagacious romance" ([10], p. 35).

Turning now to this century, a German researcher, R. Baldauf in his *Historie und Kritik*, proved on the basis of a philological argument that not only ancient, but even early medieval history was a later falsification.

An attempt to systematize the considerable amount of critical material, and to analyze historical paradoxes and duplicates from the standpoint of natural science, was carried out in the work of a remarkable scientist with encyclopaedic knowledge, a specialist in chemistry, physics, mathematics, history, a revolutionary, and public figure, honorary academician N. A. Morozov (1854–1946). He, in fact, held the opinion of de Arcilla, and believed that traditional chronology had been artificially extended [11]. It should be noted that he apparently came to this idea independently of de Arcilla and Newton. Remarkable scientific intuition and strict logical argument (astronomical, statistical, and so on) permitted Morozov to list many data in support of such a conjecture. However, his striving to dot all the i's and cross all the t's has led to poor substantiation of many of his statements; some contained factual errors, and the new chronological version as a whole (including the hypothesis regarding the falsification of antique history) was rejected, which does not at all diminish his achievements, for the problem is so complicated and many-sided that one mind alone, even if outstanding, is unable to solve it completely.

However, we definitely do not agree with the hypothesis that most literary works of antiquity are the apocrypha of the Renaissance. Nevertheless, we must investigate the problem of the dating of the ancient documents more carefully.

2. Strange Statistics of Some Astronomical Data

Let us consider some other astronomical data which can be dated using modern methods. The first example we saw above: the dating of the star catalogue in the Almagest.

Consider the three famous eclipses of Thucydides (see [12], pp. 176–179, Eclipses 6, 8, 9). They are linked into one triad by their having been described in one historical text, viz., *History of the Peloponnesian War* (Bks. ii, 27–28; iv, 51–52; vii, 18–19, 50). The descriptive characteristics of the triad, which unambiguously extracted from Thucydides' text, are the following

(1) All three eclipses were observed in the Mediterranean region, viz., in the square approximately bounded by the latitudes N 15° and 30° and longitudes E 30° and 42°.
(2) The first eclipse was solar.
(3) The second eclipse was solar.
(4) The third eclipse was lunar.
(5) The time interval between the first and second eclipses was 7 years.
(6) The time interval between the second and third eclipses was 11 years.
(7) The first eclipse occurred in summer.
(8) The first (solar) eclipse was total (since 'the stars were visible'), i.e., its phase ϕ is 12″ (maximum).
(9) The first eclipse occurred in the afternoon (local time).
(10) The second (solar) eclipse occurred at the beginning of summer.
(11) The third (lunar) eclipse occurred at the end of summer.
(12) The second eclipse occurred approximately in March.

Condition (12) clearly is not implied by Thucydides text and, therefore, is not included in the final list of conditions.

The problem arises to find a triad of eclipses completely satisfying all Conditions (1)–(11). Reference [12] gives the traditional astronomical solution, viz., 431, 424,and 413 BC. This solution was suggested by D. Petavius in the 17th century

AD. However, as was known long ago, it does not satisfy all the data of the problem. As a matter of fact, the eclipse of 431 BC was not total as required by Condition (8). It was only annular with phase 10″ for the observation zone, and could not be observed as total anywhere on the Earth's surface (*ibid.*, pp. 176–177). This important circumstance was noted by many authors, e.g., J. Zech, E. Heis, N. Struyck, G. Riccioli, F. Ginzel [12]. A considerable number of astronomcial papers were devoted to the recalculation of the phase of the eclipse of 431 BC, for which various admissible corrections were introduced into the lunar theory equations in order to make the phase close to 12″, (maximum). Thus, D. Petavius obtained $\phi=10''25$, N. Struyck 11″, J. Zech 10″38, Hoffman 10″72, and E. Heis even 7″9 (!). In the modern literature, the phase value is assumed to be 10″ (*ibid.*). F. Ginzel wrote: "The insignificance of the eclipse phase was somewhat shocking . . ." (*ibid.*).

Besides, certain other conditions were not fulfilled either, e.g., the umbra passed through the observation zone only after 17 h local times, and even after 18 h according to E. Heis, which means that Condition (9) (eclipse occurring in the afternoon) is only approximately satisfied.

Certain authors (*ibid.*; see the survey) carried out the calculation of the coordinates of bright planets, thinking that they could have been seen during the annular eclipse, in order to satisfy important Condition (8). However, the results obtained showed clearly that the planets' positions on the celestial sphere during the eclipse of 431 BC did not provide for their reliable visibility. If Venus could have been visible, then, e.g., Mars was only 3° over the horizon, while Jupiter and Saturn were below, etc. (*ibid.*, p. 177). Johnson suggested another astronomical solution for the first eclipse, viz., 433 BC, although, it soon became clear that this solution still did not satisfy the data of the problem posed, but now for other reasons [12].

An attempt to revise, in this connection, Thucydides' text itself and, in particular, Condition (8), should also be noted. However, its detailed analysis carried out at the author's request by E. V. Alexeeva (Faculty of Philology, Moscow University) showed that the eclipse characteristics were determined from Thucyduides' unambiguously. This circumstance had not been questioned earlier, though.

No other astronomical solutions in 600–200 BC, which would be more suitable than the traditional solution of 431, 424, 413 BC, seem to have been found. It is because of this fact that it has been retained in spite of the above contradiction repeatedly discussed in the literature. It must be noted that here we do not see any attempt of the astronomers to extend the time interval in the search for a solution, because all astronomers believed that the traditional chronology (which places the Peloponnesian War in the 5th century BC) is correct.

Meanwhile, the application of the formal astronomical methods and the extension of the search interval for an astronomical solution to 900 BC–AD 1600 yields two and only two exact solutions, the first having been given in paper [11], V. 4, pp. 509, 493–512, while the second was found by the author of the present work during a repeated analysis of all the eclipses from the indicated interval. Thus, the first solution is 2 August 1133*, 20 March 1140, and 28 August 1151, whereas the second is 22 August 1039, 9 April 1046, and 15 September 1057. Note that the fact of the availability of *exact* solutions is by itself already nontrivial. In both the exact solutions found, even Condition (12) is fulfilled, the one not originally included in the list of basic data. Besides, the first eclipse is total in both solutions. Consequently, we obtain an answer to all questions formulated by all the astronomers mentioned above.

This example is not unique. We will assume that an eclipse has been dated correctly if its characteristics are exactly as described in the historical source, coincide with the parameters of the real eclipse offered by the traditional chronology. N. A. Morozov suggested a method of 'impartial' dating, viz., the comparison of the characteristics of an eclipse in a primary source with those from astronomical tables. Analysis demonstrates that when not questioning the chronology of ancient events, and *a priori* regarding it as true, the astronomers often could not find a suitable eclipse in the 'desired' century, and resorted to strained interpretations (as in the example concerning the *History of the Peloponnesian War*).

It was discovered by Morozov [1], that a similar effect of 'shifting dates upwards' is extendable to those eclipses which are traditionally dated in the interval from 1000 BC to AD 500. Further investigations show that the same situation is valid, even for the AD 800–900 eclipses. Here we consider, of course, only such eclipses whose description in the historical text is sufficiently detailed (as in Thucydides' case). It is only after AD 900 that the traditional dates are satisfactorily consistent with the precise datings given by astronomy, and undoubtedly after AD 1300.

3. Moon's Elongation and R. Newton's Conjecture

Lunar theory deals, *inter alia*, with a parameter called the second derivative of the moon's elongation D''. Depending on time, the values of this parameter should be available for the past. It can be computed if the ancient eclipse data are known. The problem has been solved by the prominent British astronomer R. Newton [14]. The graph (Figure 1) he obtained turned out to be extremely surprising. Newton wrote:

> The most striking feature of Fig. 1 is the rapid decline in D'' from about AD 700 to about AD 1300 . . . This decline means that there was a 'square wave' in the osculating value of D'' . . . Such changes in D'', and such values, unexplainable by present geophysical theories . . . , show that D'' has had surprisingly large values and that it has undergone large and sudden jumps within the past 2000 yrs ([14], p. 114–115).

To explain this square wave (jump), Newton was forced to suggest that there should exist some nongravitational interactions in the Earth–Moon system [10].

* All dates given here are *anno Domini*.

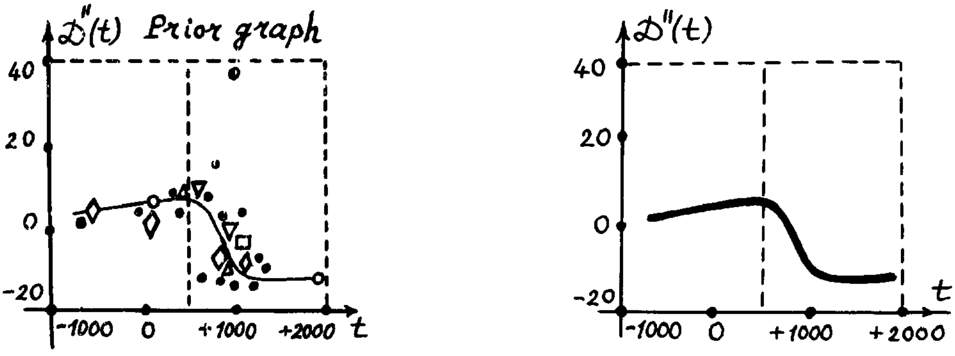


Fig. 1. R. Newton's graph demonstrating that $D''(t)$ decreases with time. See the astonishingly inexplicable jump around the first millennium AD.

These enigmatic forces do not manifest themselves in any other way, which is in itself quite unusual.

Now we shall see that there is at least one more explanation of the jump in D'' (see [15]).

Newton's computations of D'' were based upon the dates of the ancient eclipses in the traditional chronology. None of the earlier attempts to explain the strange drop (jump) in the graph of D'' questioned the accuracy of those dates. We decided to recalculate the graph D'' using the new dates of ancient eclipses, which were found by the method of formal astronomical dating (see above) without some *a priori* assumptions about the chronology of the texts. We find that the graph changes qualitatively (Figure 2). It cannot now be reliably extended to the left earlier than the 10th century AD, while in the later period, it almost coincides

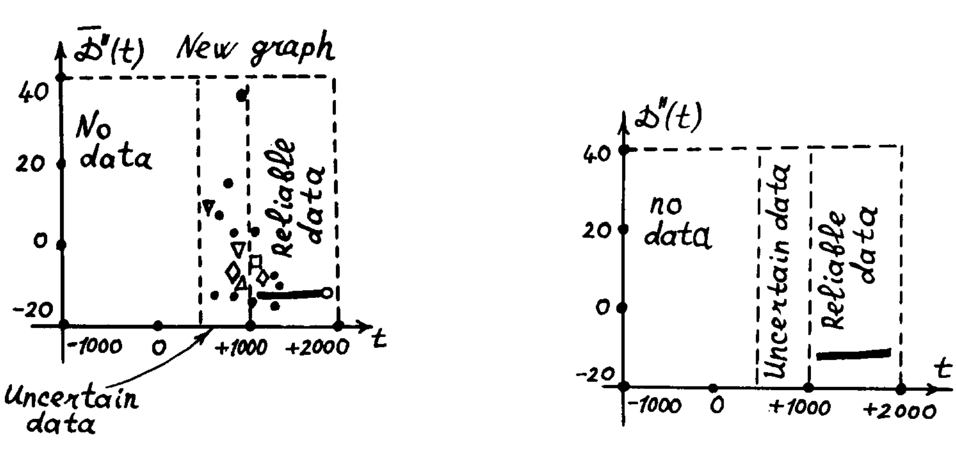


Fig. 2. The new graph of $D''(t)$, constructed on the basis of ancient eclipse dates recalculated, has no anomalies: there are simply no reliable data to extend it to the left.

with the curve already found, and is represented by almost a horizontal line. No square wave is found in the second derivative, and no mysterious nongravitational theories need be invented. We see that the new D'' varies along a smooth and nearly constant curve (horizontal in Figure 2) which oscillates about one and the same value in AD 900–1900. The parameter D'' undergoes no sharp change. It invariably approximately retains the modern value. It is interesting that the variance of the values of D'', quite insignificant in AD 900–1900, gradually increases in shifting leftwards from AD 900 to AD 400. In our opinion, this fact indicates the vagueness and insufficiency of the observational astronomical data contained in ancient historical texts describing this period. Then to the left of AD 400, the zone sets in, where reliable observational data (which may survive to the present day) are absent. See the details in [15].

4. The Dating of the Horoscopes

We now turn to the analysis of astronomical results of dating the zodiacal positions of planets, which are described in certain historical sources, viz., the so-called horoscopes. Recall that all planets are placed near the ecliptic, and their positions can be calculated similarly to the method of determining the dates of ancient eclipses. Thus, if a horoscope is described in some historical source, then, we may attempt to date it. To this end, we have to compare its description in a historical text with the calculated tabular horoscopes, and attempt to find a horoscope with the same characteristics. The seeming simplicity of this idea is greatly complicated by various side issues of a 'non-astronomical' character, which are similar to those already familiar to us (which were forced by the traditional chronology).

Morozov [11] analyzed the traditional dates of all basic horoscopes, as determined in the surviving ancient sources. Omitting the details, we can inform the reader that the same upward shift of their astronomically obtained dates resulted as occurred previously in the case of ancient eclipses. We give a typical example.

The famous Egyptologist W. Flinders Petrie discovered in 1901 in Upper Egypt (Athribis) an ancient internment dated by the traditional chronology from the 1st century BC to the 1st century AD. The internment was found to contain two graphical images of the zodiacal planets. The two horoscopes probably indicated the dates of the two tombs. The specialist Knobel [16] attempted to date the horoscopes within the *a priori* time interval from the 1st century BC to the 1st century AD. However, no exact astronomical solution was found. We make it precise that the *a priori* interval was determined by proceeding from the style and character of the inscriptions in the grave. Because of this, Knobel was forced to offer only approximate values, viz., AD 52 and 59. Knobel noted the imprecision, because the position of Venus at that time was different from its representation in the tombs. Meanwhile, the application of the formal astronomical dating method (Morozov, [11], V.6, p. 745) led to the discovery of an exact astronomical solution, viz., AD 1049 and 1065. It is important that this one is unique in the whole historical interval.

Numerous attempts by P. Laplace, J. Fourier, A. Letronne, J. Biot, and K. Helm to find a suitable solution to the horoscope represented on the 'round' and 'long' Zodiacs of the Dandarah temple in Egypt were not crowned with success ([11], V. 6, pp. 664–665, Figs. 672–673, Figs. 133, 135). The temple and horoscopes are at present dated (in the traditional chronology) by 30 BC and AD 14–37. Nevertheless, the exact astronomical solution does exist, viz., AD 568 and 540. For a detailed discussion and a list of other examples and results, see [11].

5. Empirico-Statistical Dating Methods: New Possibilities

In order to overcome the difficulties in establishing accurate chronology, one must try to view the subject from a different vantage point and create an independent methodology which is not based upon subjective impressions. Only after this is done should one proceed to analyze chronology in its entirety. In our opinion, the most worthwhile approach uses statistical analysis of various numerical characteristics of historical texts. Some new methods were suggested by the author. These methods make no claim to universality. Moreover, the results obtained by means of each individual method cannot be taken as definitive. A reasonable criterion for the validity of our results is agreement between the dates which are obtained by different methods. At present I have developed (with my colleagues) seven different methods for statistical dating, of which two will be briefly explained below.

The general scheme is as follows. First one formulates a statistical conjecture (model). Then one introduces numerical parameters which make it qualitatively possible to measure how far experiment deviates from theoretical predictions. Next, the model is checked against uncontroversial data from recent history. And finally, if this verification supports the model, then the method can be used to date the early events which interest us.

To illustrate this, we first describe the text-volume method. Suppose that the historical period from the year M to the year N in the region R is described in the text (chronicle) X, which is divided into sections ('chapters') devoted to the events of a single year. Measure the volume of each section (the number of pages or lines). Represent these numbers on a graph having a year T along the horizontal axis and the chapter volume $X(T)$ along the vertical axis. Another text Y describing the same events will generally have a different graph, reflecting the different sources, interests and styles of different chroniclers. How large are these differences between the graphs? Do the graphs have some features in common? It turns out that they do.

An essential characteristic of any graph is its number of peaks (local maxima), which in the case of our chapter-volume graphs fall on the years which are described in most detail in our chronicle. We let $C(T)$ denote the total length of all texts about the year T which were written by people living at (or soon after) the time T. This gives the 'original text graph' $C(T)$ (Figure 3). We do not know the exact form of this graph, because some original texts have been lost over time.

We now formulate a conjecture concerning the loss of information: *a greater quantity of texts remain from the years to which the greater amount of original texts were devoted*. Of course, in this form, our assertion cannot be checked, since we do not know the original text graph $C(T)$. However, there is a closely related hypothesis which can be checked. Namely, two later authors X and Y who describe the same period MN must, even if writing at very different times, rely upon the extant texts with approximately the same distribution of information about the various years. One expects that 'on the average' these chroniclers will describe in more detail the years from which the greatest volume (amount) of texts has been preserved.

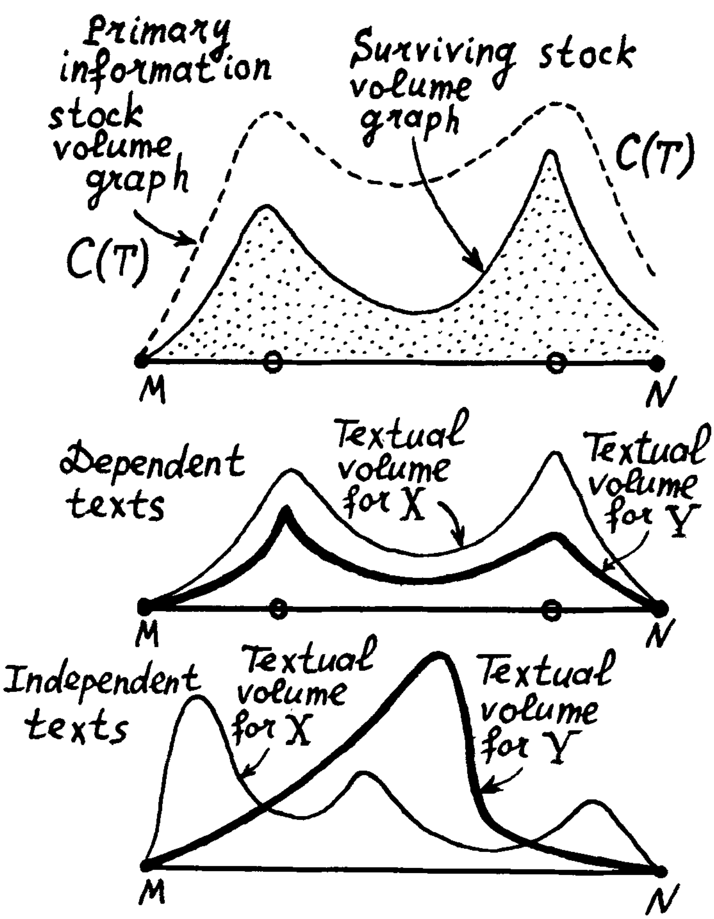


Fig. 3. Textual volume graphs in a time interval MN: (a) the primary and surviving stocks (curves should make splashes approximately at the same years); (b) the curves for dependent texts are correlated; (c) the curves for independent texts are not correlated.

The final conjecture (model) can then be stated as follows: *the chapter-volume graphs for dependent texts X and Y (i.e. which describe the same historical period and the same region and events) must have local maxima (spikes) at the same time.* In other words, the years which are described in most detail in X and Y must be the same or near one another (Figure 3). *Conversely, if the texts X and Y are independent (i.e., describe eiher different historical periods of the same length or else different regions), then their chapter-volume graphs have local maxima at different years (no correlation)* (Figure 3).

Using several hundred pairs of dependent and independent texts describing *relatively recent* history, we performed an experiment to test this principle (*maxima correlation principle*). The calculations support the conjecture (model). This enabled us to propose a method for dating texts (more precisely – the events described in the text) and for discovering dependent texts (events). For example, in order to date the events in some chronicle, one should try to find a text whose dates are known and unquestioned and whose chapter-volume graph reaches its local maxima at essentially the same points (years) as the chapter-volume graph of the undated text. Here we must identify two time intervals MN for X and AB for Y before the comparison of the graphs.

On the other hand, if we are comparing two texts neither of which has been dated, but we know that their graphs have local maxima at the same points, then with high probability, we can at least suppose that they are dependent texts, i.e., they describe essentially 'the same events'.

The application of this method to the texts which are dated in traditional chronology in the interval AD 1300–1900 (approximately) confirm the traditional dates of the events (and tend to confirm, consequently, the correctness of our method). On the other hand, the application of the same method to the texts, which are usually dated before AD 1300, gives us shocking examples of pairs of texts which are considered nowadays as independent texts (in any sense), but were fixed as dependent texts by those methods. We call such texts statistical duplicates. Our methods show the statistical dependence of such texts. Let us indicate an example of such remarkable texts.

Let us compare texts describing ancient Rome in 753–236 BC and the texts describing medieval Rome in AD 300–816. More precisely, the epoch AB from AD 300 to 816 was described, e.g., in the fundamental work of F. Gregorovius [19] *History of the City of Rome in the Middle Ages*; then the epoch CD from the year 1 to 517, since the foundation of Rome (which occurred, as is thought today, in 753 BC) is described in the following two texts. The *History of Rome* by Livy from the year 1 up to 459 since the foundation of Rome; Livy's text is then broken, and the years 459 to 517 since the foundation of Rome was 'covered' by the monograph of V.S. Sergeev, *Essays on the History of Ancient Rome* by extending Livy's text. These two can be and have been combined. This is based on the strong correlation of V. S. Sergeev's text with that of Livy with a proximity coefficient $p = 2 \times 10^{-12}$ (we do not discuss here the definition of this coefficient) (see [20]).

The computation of the coefficient $p(X, Y)$, where X is text of F. Gregorovius (medieval Rome), and Y = the sum of Livy's and Sergeev's (ancient Rome) texts, yields $p(X, Y) = 6 \times 10^{-11}$. If we drop Sergeev's text, and compare the text X' = part of the F. Gregorovius' text from AD 300 to 758 and the text Y' = part of the *History of Rome* by Livy from the year 1 to 459 since the foundation of Rome, then we compute that $p(X', Y') = 6 \times 10^{-10}$. Both of these results indicate the statistical dependence of the two 'epochs' described in the antique and medieval sources; more precisely, statistical dependence of the primary sources describing them (on which all the later texts are based). This statistical dependence is expressed vividly, and is of the same character as that between texts (of recent history) describing the same events.

All such 'epochs' (and corresponding texts) which are anomalously close from the standpoint of the coefficient $p(X, Y)$ have been marked by us on the time axis.

6. Statistically Dependent and Independent Dynasties

About fifty years ago, N. A. Morozov [11] found three pairs of ruling ancient dynasties for which the sequences of lengths (periods) of reign, represented visually on the time line, bore a striking resemblance to one another. He suggested that in each case, the two dynasties are actually reflections of a single real dynasty which 'became multiplied' as a result of a mistaken dating of the different texts describing a single sequence of events.

However, while studying this approach of Morozov, we were able to find about a dozen pairs of dynasties in recent history – where the chronology is indisputable and the dynasties are essentially different – for which the sequence of periods of reigns can be considered as 'visually close' to one another. Thus, it is not enough to rely upon a subjective impression. For this reason, we set about creating a formal qualitative method for determining whether two dynasties are merely different manifestations of a single dynasty ('dependent dynasties') or are truly distinct ('independent dynasties').

We now state the problem more precisely. Let n be a fixed integer, say $n = 15$. By a 'real dynasty' A we mean a sequence of n successive reigns (rulers) in the actual history of some region. Each chronicler who describes the dynasty A reports the length of the reign of $n = 15$ rulers, i.e., he gives us a sequence $\mathbf{a} = (a_1, \dots, a_{15})$, which we call a 'numerical dynasty'. We represent a numerical dynasty as a point in Euclidean space $\mathbb{R}^{15}$. Another chronicler who describes the same real dynasty will generally give us a *different* numerical dynasty $\mathbf{b} = (b_1, \dots, b_{15})$ – another point from $\mathbb{R}^{15}$, this is because of various types of error and imprecision in the reporting. Let $V(A)$ denote the set of all points in $\mathbb{R}^{15}$ which we obtain from all the available chronicles of the real dynasty A. How 'spread out' the cluster $V(A)$ is indicates the level of error in the chronicles.

A numerical dynasty $\mathbf{a}$ which describes the real dynasty A and a numerical dynasty $\mathbf{b}$ which describes the real dynasty B are said to be 'dependent' if $A = B$,

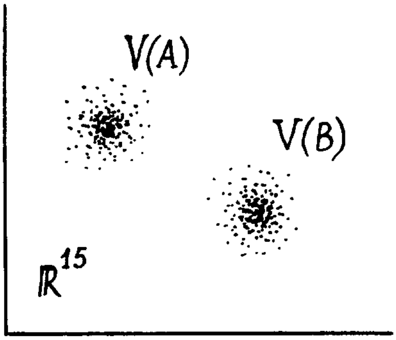


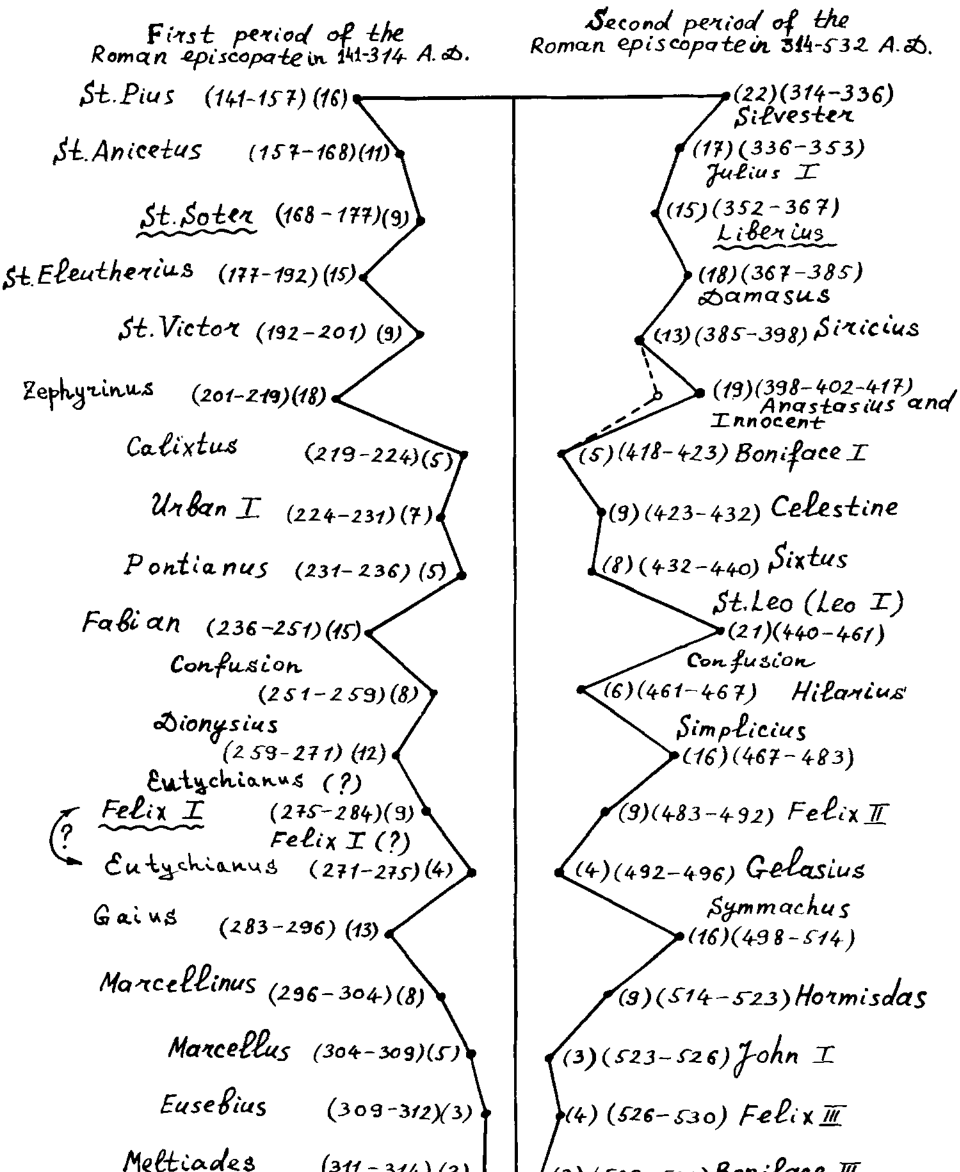
Fig. 4.

i.e., if they describe the same real dynasty. We say that **a** and **b** are 'independent' numerical dynasties if the corresponding real dynasties A and B have fewer than $n/2$ rulers in common. Besides dependent and independent pairs (**a, b**), there are also intermediate cases, when the number of rulers A and B have in common is less than n but greater than $n/2$. However, in practice, there are far fewer intermediate pairs than dependent and independent pairs, and we shall ignore the intermediate case.

Table I. (See the rule variations in [21, 22])

The 1st Roman pontificate: AD 141-314	The 2nd Roman pontificate: AD 314–532
(1) Pius I 141–157 (16)	(1) Silvester I 314–336 (18)
(2) Anicetus 157–168 (11)	(2) Julius I 336–353 (17)
(3) Soter 168–177 (9)	(3) Liberius 352-367 (15)
(4) Eleutherius 177–192 (15)	(4) Damasus I 385–398 (13)
(5) Victor I 192–201 (9)	(5) Siricius 385–398 (13)
(6) Zephyrinus 201–219 (18)	(6) Anastasius I Innocent 398–417 (19)
(7) Calixtus 219–224 (5)	(7) Boniface 418–423 (5)
(8) Urban I 224–231 (7)	(8) Celestine I 423–432 (9)
(9) Pontianus 231–236 (5)	(9) Sixtus III 432–440 (8)
(10) Fabian 236–251 (15)	(10) Leo I 440–461 (8)
(11) Confusion 251–259 (8)	(11) Confusion, Hilarius 461–467 (6)
(12) Dionysius 259–271 (12)	(12) Simplicius 467–483 (16)
(13) Felix I (or Eutychianus ?) 275–284 (9)	(13) Felix III 483–492 (9)
(14) Eutychianus (or Felix I ?) 271–275 (4)	(14) Gelasius 492–496 (4)
(15) Gaius 283–296 (7)	(15) Symmachus 498–514 (16)
(16) Marcellinus 296–304 (8)	(16) Hormisdas 514–523 (19)
(17) Marcellus I 304–309 (5)	(17) John I 523–526 (3)
(18) Eusebius 309–312 (3)	(18) Felix IV 526–530 (4)
(19) Meltiades 311 314 (3)	(19) Boniface III 530–532 (2)

We are now ready to state a conjecture (model) which, if valid, will help us in dating ancient events. We call this conjecture the 'principle of small distortions'. Roughly speaking, it says that the chroniclers' errors were relatively small, i.e., the cluster $V(A)$ is tightly bunched together relative to the distance between $V(A)$ and $V(B)$ when A and B are independent real dynasties (Figure 4). More precisely,



Sources: [21, 22].

Fig. 5.

we conjecture that *there exists a natural distance function* $d(\mathbf{a}, \mathbf{b})$ *on* $\mathbb{R}^{15}$ *such that* $d(\mathbf{a}, \mathbf{b})$ *is very small when a and b are dependent and has a larger order of magnitude when a and b are independent.*

First, from approximately 300 historical chronicles, annals, tables, and other primary sources, we compiled a data file of all 15-member numerical dynasties in Europe, the Mediterranean, the Middle East and Egypt between 4000 BC and the 19th century AD. We were then able to find a distance function $d(\mathbf{a}, \mathbf{b})$ on $\mathbb{R}^{15}$ with the following property. When applied to numerical dynasties a, b from the recent period (since the 13th century), where there is no controversy over traditional chronology, we obtain $d(a, b) < 10^{-8}$ for any dependent pair of dynasties and $d(\mathbf{a}, \mathbf{b}) > 10^{-3}$ for any independent pair. For more details see [20]. Thus, the conjectured principle of small distortions is supported by a vast array of historical evidence. It is reasonable to suppose that this metric $d(\mathbf{a}, \mathbf{b})$ can be

Table II

(A) Carolingians, Charlemagne's Empire in the 6–9th centuries AD	(B) A strand (i.e., some part) from the Roman Empire in the 3rd–4th centuries AD.
The average shift with respect to the rule endings equals 359.6 years, which coincides with the first basic rigid 360–year shift, making coincident the left column with the right.	
(A) Roman Empire in the 10–13 centuries AD	(B) Roman Empire in the 4–6th centuries AD
The average shift with respect to the rule endings is 723 years, which is close to the rigid 720-year shift making the left column coincident with the right.	
(A) Holy Roman Empire in the 10–13th centuries AD. Start of the Saxon Dynasty in 911AD.	(B) Empire of the Habsburgs in 13–17th centuries AD. Start of the Austrian duchy in 1273.
Overlapping can be obtained under a 362-year rigid shift	
(A) Holy Roman Empire in Italy in 10–13th centuries AD. Lasts for 292 years from 962 or 964 to 1254.	(B) Roman Empire from the 1st century BC to the 3rd century AD in Italy. Lasts for 299 years from 82 BC to AD 217
Overlapping occurs under the rigid upward shift through 1053 years.	
(A) Regal Rome, 753–500 BC according to Livy.	(B) A Roman Empire strand in the 3rd–4th centuries AD
Here the shift amounts to 1050 years.	
(A) Roman Empire from 82 BC to the 3rd century AD (Figure 6).	(B) A strand from the Roman Empire in the 3rd –6th centuries AD.
(A) Byzantine Empire during the period AD 527–829.	(B) Byzantine Empire in AD 829–1204.
The left and right columns can be made coincident by a rigid basic shift through about 340 years. The same shift (!) makes the other two Byzantine Empires coincident.	
(A) Byzantine Empire AD 1204–1453.	(B) Byzantine Empire, the strand from Basil I until John III. AD 867–1143.
These two empires can also be made to coincide by a 340-year shift.	

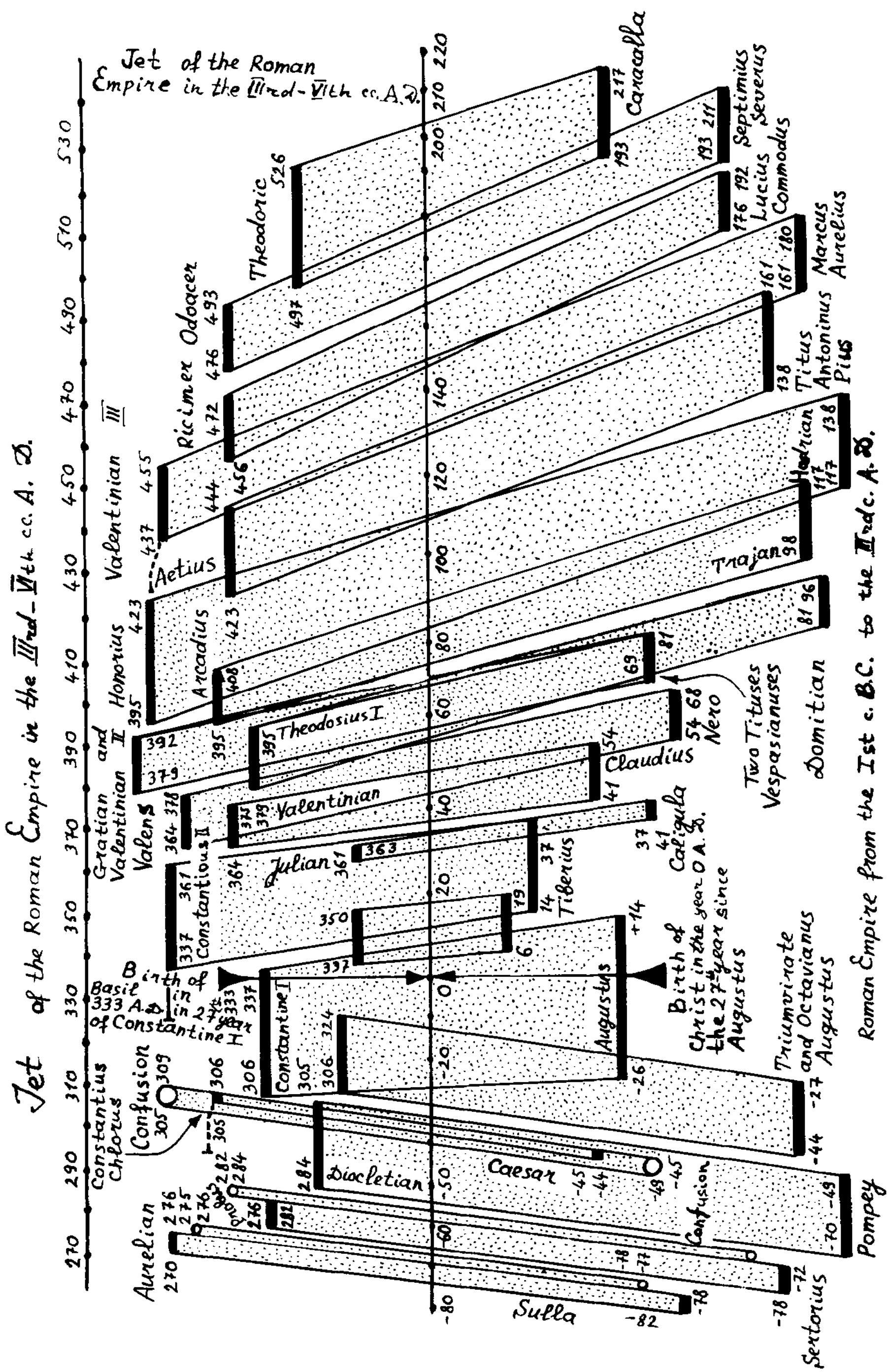

Jet of the Roman Empire in the IIIrd–VIth cc. A. D.
Jet of the Roman Empire in the IIIrd–VIth cc. A.D.
Roman Empire from the Ist c. B.C. to the IIIrd c. A. D.
Theodoric
Odoacer
Ricimer
Valentinian III
Aetius
Honorius
Arcadius
Theodosius I
Gratian and Valentinian II
Valens
Valentinian
Julian
Constantius II
Birth of Basil 333 A.D. in 27th year of Constantine I
Constantine I
Confusion
Constantius Chlorus
Diocletian
Aurelian
Caracalla
Septimius Severus
Lucius Commodus
Marcus Aurelius
Titus Antoninus Pius
Hadrian
Trajan
Domitian
Two Tituses Vespasianuses
Nero
Claudius
Caligula
Tiberius
Birth of Christ in the year 0 A.D. in the 27th year since Augustus
Augustus
Triumvirate and Octavianus Augustus
Caesar
Confusion
Pompey
Sertorius
Sulla

Fig. 6.

applied to numerical dynasties from the more distant past as a criterion for determining statistical dependence and independence.

The application of the method to historical data traditionally believed to belong to centuries earlier than the 13th century AD unexpectedly led to the discovery of dynastic pairs **a** and **b** regarded as independent in every sense, but for which the proximity coefficient $d(\mathbf{a}, \mathbf{b})$ is of the same order as for necessarily dependent dynasties, and does not exceed 10^{-8}. Below, we give examples of such dynastic pairs. It means that they are, probably, dependent, and are statistical duplicates.

Remark. In Table I, the rule periods are considered as approximative. The details and value of admissible error for the rule periods can be found in [20].

The graphical representation can be found in Figure 5.

Let us list some more examples (without details) of statistically dependent pairs discovered by computer experiments (Table II).

7. Statistical Duplicates

Sometimes statistical methods such as those described in the last two sections seem to indicate that certain chronicles which were traditionally assumed to be describing completely different historical segments, are actually about the same sequence of events. But we do not want to formulate now some final hypothesis and, to avoid confusion in terminology, in what follows we shall simply use the word 'statistical duplicate' or 'duplicate'.

The time has come to state our central problem: Find, if possible, all duplicates in ancient and medieval chronology. Is there something systematic in the distribution of such duplicates on the time axis?

8. Global Chronological Diagram

Before undertaking a systematic analysis of historical texts with the goal of discovering and listing all statistical duplicates, we had to put together as complete a table as possible of the events in the ancient and medieval history of Europe, the Mediterranean, Egypt, and the Middle East – of course, in the traditional chronology. To do this, we combined the information from 15 chronological tables and 228 primary sources (such as chronicles). Together, these texts describe practically all important events between 4000 BC and AD 1800. We then represented all of the data graphically on the plane. Each historical period with all of its key events found its place on the time axis. Every event (its date) corresponds to a point or horizontal segment, depending on its duration; the endpoints of the segments are the beginning and end of the event (for example, a ruler's reign). Events which occur at the same time are placed on top of one another, so as to avoid the confusion that would result from superimposing them.

In this way, we constructed a maximally complete chronological and graphical table, which we call a 'global chronological diagram' (GCD).

In order to determine what events took place in a certain year according to traditional chronology, we simply draw a vertical line through that year in the GCD and take all the events which intersect the line. Let us give us an example, a small part of our GCD:

474 491

Zeno 491 518

457 Leo I 474

Anastasius

476 493

Odoacer

Our statistical methods for dating events and recognizing duplicates were applied to the large quantity of historical data in the GCD table. For example, to apply the text-volume method, we divided the entire time period covered by the GCD into segments and for each time segment in each region we constructed the chapter-volume graph for all primary sources which describe the period. We then compared pairs of graphs from different periods. After performing a rather substantial computer experiment, involving the analysis of hundreds of texts, we unexpectedly discovered pairs of periods which in traditional history are assumed to be independent (in all senses of the word) but which have graphs which are extremely close. Let us give an example. As we saw above, the chapter-volume graph for the primary sources which describe Roman history from 753 to 236 BC has its local maxima at essentially 'the same points' as the analogous graph for Roman history from AD 300 to AD 816. Of course, here we must first align the two 500-year time inervals. The same statistical parallelism was also revealed by other methods. See details in [20].

It turned out that the GCD has many duplicates, i.e., many pairs of texts and corresponding historical periods whose primary sources are as close as are pairs of texts which are known to describe the same period (the same events). It should be emphasized that the results obtained using different quantitative methods agreed with one another.

9. A Four-Fiber Textbook of Ancient History

The Global Chronological Diagram represent the modern version of the traditional 'textbook' of ancient and medieval history and chronology (in traditional datings). We investigate the inner hidden structure of the GCD, i.e., of the textbook of ancient history (roughly speaking, this 'textbook' is based on Scaliger's version of global chronology). Our calculations (see above) show that the 'modern textbook' contains a very interesting system of statistical duplicates. How does one understand the origin of such duplicates and their distribution along the time axis (in the 'textbook')?

In order to make it easier to visualize the set of duplicate periods that were discovered, we have used the same letters and geometric symbols (arbitarily chosen) to denote them on the diagram. As you can see in Figure 7, some letters are repeated many times (for example, T occurs 11 times, and C occurs 4 times), indicating many duplications of a single period.

The length of geometrical figures in Figure 7d gives the duration of the corresponding period of time. For example, the black triangles labeled T correpond to intervals of about 20–30 years, and the rectangles C correspond to 300-year periods. Some time intervals on the GCD are covered with several figures. The time period from about AD 300–550, for instance, has four rectangles O, K, C, P on top of one another. This means that the corresponding part of the 'textbook of ancient history' is foliated into the sum of four fibers indicated by O, K, C, P. Each of them is some part of the modern textbook. In other words, 'each page' of the modern textbook (devoted to the events from the interval AD 300–550) is divided into four pieces belonging to the parts O, K, C, P. Roughly speaking, we divide each page into four parts and mark them by O, K, C, P.

The most important point is that the rather complicated structure of the GCD can be obtained in a natural way by means of a very curious procedure. We can pick out from the map four rows $C1$, $C2$, $C3$, $C4$ (Figure 7) such that if we glue them together vertically, then we recover our GCD with its lettered time periods. What is most unexpected is that these four rows have practically the same sequence of letters and symbols. The four duplication-fibers differ only in their location on the time axis. That is, the row $C2$ differs from $C1$ simply by a backward time shift of 333 years, the third row can be obtained from $C1$ by means of a 1053-year shift, and $C4$ can be obtained by a 1778-year shift.

Speaking loosely, *we might say that the commonly accepted 'textbook' of ancient and medieval European, Mediterranean, Egyptian and Middle Eastern history is a fibered (layered) chronicle obtained by gluing together four nearly identical copies of a shorter chronicle C1. The other three chronicles are obtained from C1 by redating and renaming the events described in them; we rigidly move C1 in its entirety backwards in time by 333, 1053 and 1778 years (approximately). Thus, the full 'textbook' (=GCD) can be reconstructed from its smaller part; namely, from the 'chronicle' C1.*

Remark. Observe that not all duplications disappear when we pass to the component $C1$: we still have a repetition of P and two duplicates of T. But a large proportion of the duplication in the textbook GCD can be accounted for simply by postulating that the 'events' in the three rows, $C2$, $C3$, $C4$ should all be redated according to the rigid shifts indicated.

And there is something further that should be noted: almost all of the information in the chronicle row $C1$ is concentrated to the right of AD 960. The

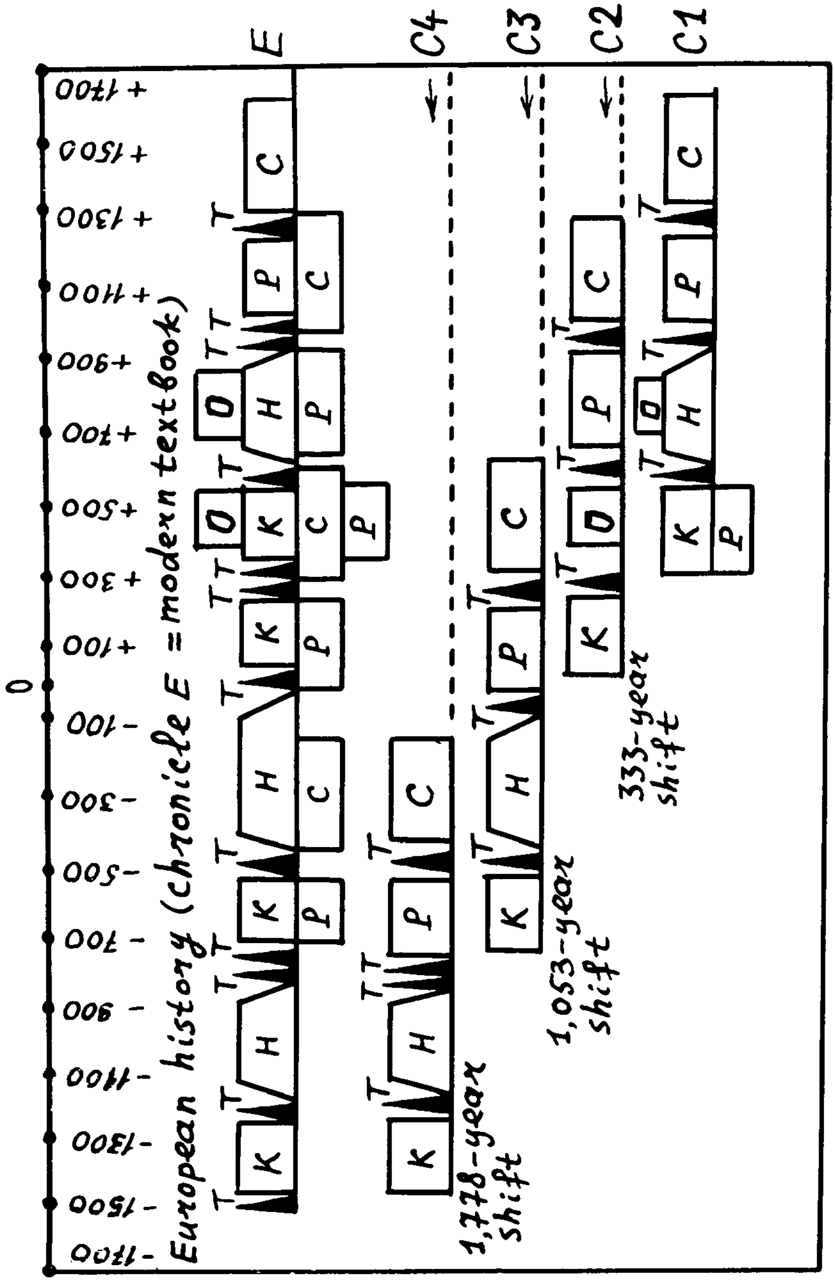

European history (chronicle E = modern textbook)
E
C4
C3
C2
C1
1,778-year shift
1,053-year shift
333-year shift
-1700
-1500
-1300
-1100
-900
-700
-500
-300
-100
0
+100
+300
+500
+700
+900
+1100
+1300
+1500
+1700

Fig. 7a.

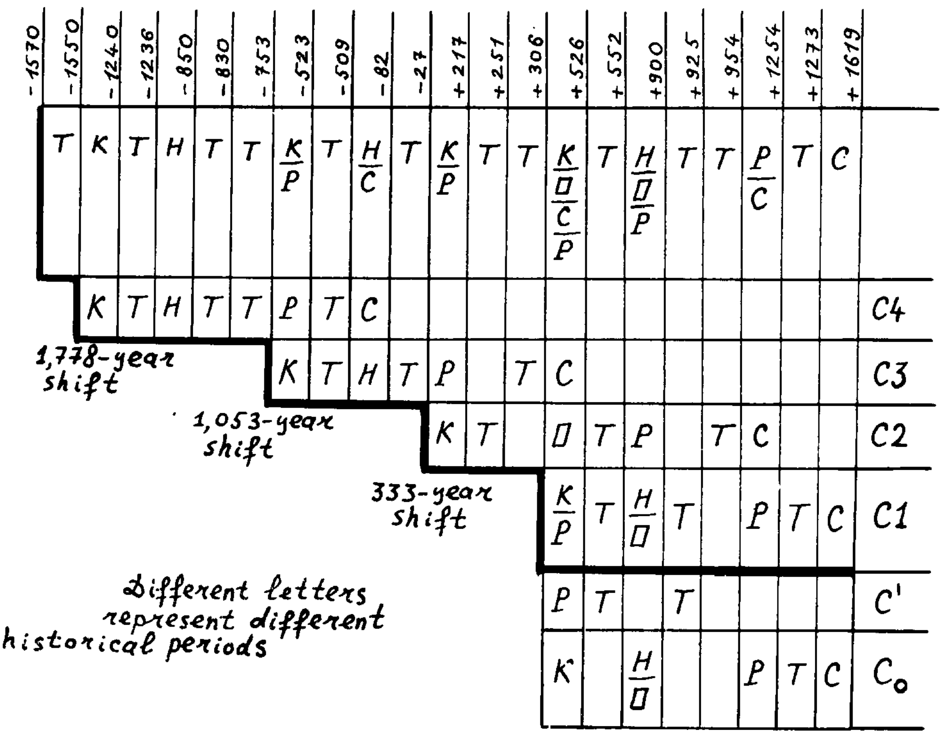


Fig. 7. Different letters represent different historical periods.

heaviest concentration of data is in the periods P, T and C, while the periods K, H and O between AD 300 and AD 960 contain very little written information about the events of this period. Thus, despite the spread out appearance of the GCD, in reality we have extensive historical records only from the three segments to the right of AD 960.

This decomposition of the 'modern textbook of ancient history' into the 'sum' of four identical fibers is our main statistical result based on computer investigations.

10. Interpretations of the Main Result and Hypothesis

Generally speaking, the main result of our work has a formal statistical character, and nothing more. But it is natural to try to understand the origin of such important fact. This formal decomposition of the history 'textbook' into the sum of four chronicles can be *interpreted* in various ways.

The first possibility is to say that the periodicity is accidental. However, one can compute (by mathematical methods) that the probability of such coincidence is extremely small (in the case of dynasties, this number is around 10^{-12}).

A second interpretation is that we do not have enough written records of certain periods in ancient history, and this makes it difficult to apply statistical methods.

Finally, there is a third *interpretation* which we believe merits serious consideration. It says that the currently accepted global chronology before the 13th century AD is in need of modification, and, in fact requires some radical changes. Conjecture: we have to redate certain blocks of ancient events which until now have been placed in very ancient times. To do this, we must take out rows $C4$, $C3$, $C2$ from our chronological diagram (from the textbook) and move them forward in time, as indicated above. After this procedure, one finds that the known written history of Europe, the Mediterranean, etc. becomes much shorter: most of the events which are traditionally dated earlier than the 10th century AD, turn out to be in the time interval from the 10th to 17th centuries AD. Revising chronology in this way, one finds that many of the old paradoxes in traditional dating disappear.

However, we categorically disagree with the suggestion of N. A. Morozov and some of his predecessors that our information about antiquity is a fabrication (some fantasy) of later chroniclers. The results obtained by means of the new statistical dating methods show that almost all surviving ancient documents (of antiquity or the Middle Ages) are authentic and written for the purpose of perpetuating real events rather than leading future historians astray. More than that, the discovered decomposition of the GCD (and consequently, some new 'statistical chronology') justify the authenticity of many documents (e.g., the Almagest of Ptolemy), i.e., many of the documents regarded today as adulterated, turn out to be originals which are extremely consistent with the new statistical chronology, following from the decomposition of the 'textbook' into the sum of three shifts. In our opinion, practically everything described in the old document 'did, in fact, occur'. The problem is *when and where*?

Of course, the research reported on here cannot claim to establish any final conclusions, especially since we have used purely mathematical methods to analyze what is really very complicated, multifaceted and sometimes subjectively embellished material from the historical chronicles. Without doubt, a complete treatment of the problem requires a combination of different methods, including those of pure history, archaeology, philology, physics, chemistry, and, finally, mathematics, which, as the reader has seen, is capable of giving us a new vantage point from which to view the problem of chronology.

11. Program of Investigation of Global Ancient Chronology

The reader who is interested in this problem can read the author's book *Empirico-Statistical Methods for Analysis of Narrative and Numerical Sources with Applications to the Problems of Ancient Chronology* (the recognition and dating of dependent texts, new statistical ancient chronology, statistics of ancient astronomical records) which is shortly to be published by Kluwer Academic Publishers. This book is devoted to an attempt to realize the program of the analysis of global chronology. This topic includes the following aspects:

(1) The investigation of the Moon's elongation and R. Newton's conjecture. Statistics of astronomical information in ancient documents.

The parameter D''. Statistical regularities in the distribution of the dates of ancient eclipses. The strange character of the graph of D''. Method of formal astronomical dating of eclipses decribed in historical texts. Morozov's analysis of the whole list of records containing information about astronomical events: solar and lunar eclipses. Redating most of the 'well-described' ancient eclipses. The shift to the right of the dates of ancient eclipses under formal astronomical dating. Important examples of the eclipses described in the texts of Thucydides, Livy etc. Different groups of the eclipses which involve different values of the rigid shifts of their dates. Systematization of such shifts. Three main shifts of the dates of eclipses. The analysis of the complete list of star flares and the distribution of their dates. The astronomical dating of ancient horoscopes.

(2) Information functions of the historical (narrative) texts and related statistical regularities.

The notion of the general information function. Some natural models describing the evolution of information contained in historical texts. Important examples are the volume functions of historical text; the local maxima of the graph of the volume function; a theoretical model (statistical hypothesis) describing the distribution of local maxima for the textual volume function. Other important related aspects include: the correlation of the maxima principle; mathematical formalization and testing of the principle; analysis of important ancient texts; (Livy, Herodotus, etc.); the analysis of 'written biographies' contained in ancient texts; a new notion for questionnaire forms, or 'formal biography', which formalizes and possibly separates all basic facts from the 'written biography' of some historical person; statistical method of comparison of sequences of 'formal biographies' of the rulers (in ancient chronicles); 'formal biographies' of the rulers of regal Rome and Byzantine emperors.

(3) A method of duplicate recognition. Statistical analysis of the ancient dynasties of the rulers.

Real dynasties and numerical dynasties. A virtual variation of a dynasty. Complete list of all numerical dynasties from the history of Europe, the Mediterranean region, Near East and Egypt from 4000 BC to AD 1800. Dependent and independent dynasties. Statistical analysis of the basic historical dynasties: bishops and popes in Rome, Saracens, high priests in Judaea, Greeks in Bactria, exarchs in Ravenna, all dynasties of the Pharoahs and other Egyptian rulers, dynasties of the Byzantine Empire, Roman Empire, Spain, Russia, France, Italy, the Ottoman Empire, Scotland, Lakedaemon, Germany, Sweden, Denmark, Israel, Bablylonia, Syria, Sicion, Judaea, Portugal, Parthia, the Bosphorus, Macedonia, Poland and England. The discovery of pairs of historical dynasties earlier regarded as independent in all senses, however, the value of the coefficient $d(\mathbf{a}, \mathbf{b})$ for them is the same as for pairs of necessarily dependent dynasties.

(4) New empirico-statistical procedure for text ordering. The recognition of statistical duplicates.

This method allows us to find a chronologically correct order of separate fragments of historical texts, and discover among them various duplicates, or repetitions, i.e., parts describing the same events. The notion of 'chapter-generation'. The important frequency damping principle. The frequency duplication principle and its applications to the problems of ancient chronology. The square matrix of frequencies of ancient names. Application to the analysis of Byzantine history and chronology.

(5) N. A. Morozov, his investigations, the fundamental work *Christ*. I. Newton and his investigations in ancient chronology.

(6) The global Chronological Diagram – the modern 'textbook of ancient and medieval history' (in traditional datings). Four fibers in this 'textbook'. Three rigid shifts by 330, 1050 and 1800 years (approximately) corresponding to these fibers. Statistical decomposition of the GCD into the sum of four fibers.

(7) Different interpretations of the main statistical result (four-fiber structure of the 'historical textbook'). Hypothesis about the origin of three rigid shifts in ancient chronology.

(8) The chronology of J. Scaliger, D. Petavius, Eusebii Pamphili, Theophilus, etc. Different dates for the biblical creation of the world. Roman chronology and hypercriticism, T. Mommsen, de Arcilla, J. Hardouin, R. Baldauf, I. Newton, P. Hochart, J. Ross. Analysis of the discovery of the works of Quintilian, Valerius Flaccus, Marcellus, Probus, Cicero, Lucretius, Petronius, Livy, Plautus, Herodotus, Tacitus, Suetonius, Vitruvius, Diophantus, Archimedes etc. The chronological history of religious texts.

(9) Ancient geography and medieval geography. Ancient maps and medieval maps. The medieval ideas of chronological scheme of the world history.

(10) Astronomical analysis of the Almagest (Ptolemy). Redating of the Almagest. R. Newton's book *The Crime of Claudius Ptolemy*. Statistical analysis of the star catalogue in the Almagest. Analysis of other ancient star catalogues.

(11) Analysis of classical historical sources and monument dating methods. The difficulties of the radiocarbon method. W. Libby and Vladimir Milojčič's critique of radiocarbon dating methods. Theoretical and practical accuracy of radiocarbon dating methods. Another physical method of dating.

(12) The detailed statistical analysis of Roman chronology and Roman history.

(13) The detailed statistical analysis of Greek chronology and Greek history.

(14) Statistical parallels between ancient and medieval Roman history. Parallels between ancient and medieval Greek history.

(15) Medieval Europe and the ancient world. Parallels and duplicates. Three chronological shifts.

(16) Tables of the main statistical 'biological' parallelisms between ancient and medieval events.

(17) Hypothesis on a new 'short statistical chronology'. The main chronological basic points of new statistical chronology.

(18) Unsolved problems. Further approaches to the analysis of ancient chronology.

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